Statistical inference for equality of two normal distributions
The statistical inference for equality of two normal distributions is translated into inferences for equality of means and equality of standard deviations of both distributions:

$$\mathrm{H}(F_1(X) = F_2(X)) \rightarrow \mathrm{H}(m_1 = m_2, \sigma_1 = \sigma_2) \tag{1}$$

Since the mean and the standard deviation are independent of each other, the hypothesis $\mathrm{H}(m_1 = m_2, \sigma_1 = \sigma_2)$ is valid if and only if both of the hypothesis $\mathrm{H}(m_1 = m_2)$ and $\mathrm{H}(\sigma_1 = \sigma_2)$ are valid. Therefore, the equality of means and the equality of standard deviations can be tested separately.

The equality of means is tested by stating the null hypothesis $\mathrm{H}_0(m_1 = m_2)$, which is translated into $\mathrm{H}_0(m_1 - m_2 = 0)$. A hypothesis test on the *difference of the means* is used in cases a) with known variances and any n_1 and n_2 or b) with unknown variances and $n_1 > 30$, $n_2 > 30$, where we estimate $\sigma_1^2 = S_1^2$, $\sigma_2^2 = S_2^2$:

H_0	H_1	test statistics	rejection region for H_0
$m_1 - m_2 = 0$	$m_1 - m_2 \neq 0$	$Z = \frac{\langle X_1\rangle - \langle X_2\rangle}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	$z < -z_{\alpha/2} \cup z > z_{\alpha/2}$

and c) with unknown, but similar, variances and any n_1, n_2:

H_0	H_1	test statistics	rejection region for H_0
$m_1 - m_2 = 0$	$m_1 - m_2 \neq 0$	$T = \frac{\langle X_1\rangle - \langle X_2\rangle}{S_p\sqrt{1/n_1 + 1/n_2}}$	$t < -t_{n_1+n_2-2;\alpha/2} \cup t > t_{n_1+n_2-2;\alpha/2}$

The parameter S_p in the test statistics is defined by the sample standard deviations:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}\,.$$

The equality of standard deviations is tested by stating the null hypothesis $\mathrm{H}_0(\sigma_1 = \sigma_2) = \mathrm{H}_0(\sigma_1^2 = \sigma_2^2)$, which is translated into $\mathrm{H}_0(\sigma_1^2/\sigma_2^2 = 1)$. A hypothesis test on the *ratio of the variances* is used:

H_0	H_1	test statistics	rejection region for H_0
$\frac{\sigma_1^2}{\sigma_2^2} = 1$	$\frac{\sigma_1^2}{\sigma_2^2} \neq 1$	$F = \frac{S_1^2}{S_2^2}$	$f < 1/f_{n_2-1,n_1-1;\alpha/2} \cup f > f_{n_1-1,n_2-1;\alpha/2}$

For defining the rejection region critical value, the following property of the F distribution is used:

$$f_{n_1,n_2;1-\alpha} = \frac{1}{f_{n_2,n_1;\alpha}}\,.$$