Parametric hypothesis testing

Statistical hypothesis is a statement about some parameter or probability distribution of one or more populations. When the hypothesis is about a parameter it is called *parametric*, when it is about a probability distribution it is called *non-parametric*. The hypothesis testing is the decision-making procedure about the hypothesis. In this set the parametric hypothesis testing is discussed.

The statement under test is called the *null hypothesis* $\mathrm{H}_0()$ and the contradictory statement is called the *alternative hypothesis* $\mathrm{H}_1()$. The null hypothesis always states that a parameter (e.g. population mean) is equal to some value, e.g.:

$$\mathrm{H}_0(m = 20)\,. \tag{1}$$

The alternative hypothesis states that the parameter is either non-equal (two-sided alternative hypothesis), less than (left one-sided) or greather than (right one-sided alternative hypothesis) that value, e.g.:

$$\mathrm{H}_1(m \neq 20) \qquad \text{or} \qquad \mathrm{H}_1(m < 20) \qquad \text{or} \qquad \mathrm{H}_1(m > 20)\,. \tag{2}$$

A hypothesis testing procedure consists of eight steps:

1. From the problem context, identify the parameter of interest.
2. State the null hypothesis, $\mathrm{H}_0()$.
3. Specify an appropriate alternative hypothesis, $\mathrm{H}_1()$. It is either two-sided or left/right one-sided.
4. Choose a significance level α. Usually, $\alpha = 0.05$ or 0.01.
5. Determine an appropriate test statistic.
6. State the rejection region for the statistic.
7. Compute any necessary sample quantities, substitute these into the equation for the test statistic, and compute that value.
8. Decide whether or not $\mathrm{H}_0()$ should be rejected and report that in the problem context.

in testing any statistical hypothesis, four different situations determine whether the final decision is correct or in error:

	$\mathrm{H}_0()$ is true	$\mathrm{H}_0()$ is false
$\mathrm{H}_0()$ is rejected	type I error	no error
$\mathrm{H}_0()$ is accepted	no error	type II error

The **type I error** is made when a true hypothesis is rejected. The probability of making a type I error is equal to significance level α and is chosen in advance:

$$P\,(\text{type I error}) = \alpha\,. \tag{3}$$

The **type II error** is made when a false hypothesis is not rejected. The probability of making a type II error is denoted by β and cannot be chosen in advance since it depends on the properties of the tested population.

$$P\,(\text{type II error}) = \beta\,. \tag{4}$$

Hypothesis test on the **mean** of a) normally distributed population with known variance and any value of sample size n or b) any population distribution with unknown variance and $n > 30$, where σ^2 is estimated by S^2:

H_0	H_1	test statistics	rejection region for H_0
	$m < m_0$		$z < -z_\alpha$
$m = m_0$	$m \neq m_0$	$Z = \frac{\langle X\rangle - m_0}{\sigma/\sqrt{n}}$	$z < -z_{\alpha/2} \cup z > z_{\alpha/2}$
	$m > m_0$		$z > z_\alpha$

Hypothesis test on the **mean** of normally distributed population with unknown variance and $n < 30$:

H_0	H_1	test statistics	rejection region for H_0
	$m < m_0$		$t < -t_{n-1;\alpha}$
$m = m_0$	$m \neq m_0$	$T = \frac{\langle X\rangle - m_0}{S/\sqrt{n}}$	$t < -t_{n-1;\alpha/2} \cup t > t_{n-1;\alpha/2}$
	$m > m_0$		$t > t_{n-1;\alpha}$

Hypothesis test on the **variance** of normally distributed population:

H_0	H_1	test statistics	rejection region for H_0
	$\sigma^2 < \sigma_0^2$		$\chi^2 < \chi^2_{n-1;1-\alpha}$
$\sigma^2 = \sigma_0^2$	$\sigma^2 \neq \sigma_0^2$	$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$	$\chi^2 < \chi^2_{n-1;1-\alpha/2} \cup \chi^2 > \chi^2_{n-1;\alpha/2}$
	$\sigma^2 > \sigma_0^2$		$\chi^2 > \chi^2_{n-1;\alpha}$

Hypothesis test on the **proportion of the population** if the binomial distribution can be approximated by a normal:

H_0	H_1	test statistics	rejection region for H_0
	$p < p_0$		$z < -z_\alpha$
$p = p_0$	$p \neq p_0$	$Z = \frac{p - p_0}{\sqrt{p_0(1-p_0)/n}}$	$z < -z_{\alpha/2} \cup z > z_{\alpha/2}$
	$p > p_0$		$z > z_\alpha$

Hypothesis test on the **sum (difference) of the means** of a) normally distributed populations with known variances and any n_1 and n_2 or b) any population distributions with unknown variances and $n_1 > 30$, $n_2 > 30$ where it is estimated $\sigma_1^2 = S_1^2$, $\sigma_2^2 = S_2^2$):

H_0	H_1	test statistics	rejection region for H_0
	$m_1 \pm m_2 < \Delta_0$		$z < -z_\alpha$
$m_1 \pm m_2 = \Delta_0$	$m_1 \pm m_2 \neq \Delta_0$	$Z = \frac{[(\langle X_1\rangle \pm \langle X_2\rangle) - \Delta_0]}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	$z < -z_{\alpha/2} \cup z > z_{\alpha/2}$
	$m_1 \pm m_2 > \Delta_0$		$z > z_\alpha$

Hypothesis test on the **sum (difference) of the means** of normally distributed populations with unknown, but similar, variances and any n_1, n_2:

H_0	H_1	test statistics	rejection region for H_0
	$m_1 \pm m_2 < \Delta_0$		$t < -t_{n_1+n_2-2;\alpha}$
$m_1 \pm m_2 = \Delta_0$	$m_1 \pm m_2 \neq \Delta_0$	$T = \frac{[(\langle X_1\rangle \pm \langle X_2\rangle) - \Delta_0]}{S_p\sqrt{1/n_1 + 1/n_2}}$	$t < -t_{n_1+n_2-2;\alpha/2} \cup t > t_{n_1+n_2-2;\alpha/2}$
	$m_1 \pm m_2 > \Delta_0$	$S_p^2 = \left((n_1-1)S_1^2 + (n_2-1)S_2^2\right)/(n_1+n_2-2)$	$t > t_{n_1+n_2-2;\alpha}$

Hypothesis test on the **sum (difference) of the proportion of the population**, if the binomial distributions can be approximated by normal:

H_0	H_1	test statistics	rejection region for H_0
	$p_1 \pm p_2 < \Delta_0$		$z < -z_\alpha$
$p_1 \pm p_2 = \Delta_0$	$p_1 \pm p_2 \neq \Delta_0$	$Z = \frac{(p_1 \pm p_2) - \Delta_0}{\sqrt{p_1(1-p_1)/n_1 + p_2(1-p_2)/n_2}}$	$z < -z_{\alpha/2} \cup z > z_{\alpha/2}$
	$p_1 \pm p_2 > \Delta_0$		$z > z_\alpha$

Hypothesis test on the **variance ratio** of normally distributed populations:

H_0	H_1	test statistics	rejection region for H_0
	$\sigma_1^2/\sigma_2^2 < \Delta_0$		$f < 1/f_{n_2-1,n_1-1;\alpha}$
$\sigma_1^2/\sigma_2^2 = \Delta_0$	$\sigma_1^2/\sigma_2^2 \neq \Delta_0$	$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$	$f < 1/f_{n_2-1,n_1-1;\alpha/2} \cup f > f_{n_1-1,n_2-1;\alpha/2}$
	$\sigma_1^2/\sigma_2^2 > \Delta_0$		$f > f_{n_1-1,n_2-1;\alpha}$