Problems for exercises in Random phenomena course - 5th set

1. Synthetic fibers used to make carpets have normally distributed tensile strength with an average of 520 kPa and a standard deviation of 25 kPa.
   (a) What is the probability that the average tensile strength of a random sample of six fibers is greater than 522 kPa? R: $P = 0.422$
   (b) What is the probability of the first problem case if the sample size is increased from six to 50 fibers? R: $P = 0.286$

2. Using the method of moments find the estimator for the parameter λ of the exponential probability distribution displaced for x_0: $f(x) = \lambda\, \mathrm{e}^{-\lambda(x-x_0)}$. R: $\lambda = 1/\left(\langle X\rangle - x_0\right)$

3. Using the method of maximum likelihood find the estimator for the parameter λ of the Poisson probability distribution. R: $\lambda = \langle X\rangle$

4. Using the method of maximum likelihood find the estimator for the parameter q of the probability distribution with a probability density function $f(x) = (q+1)x^q$ for $0 \leq x \leq 1$. R: $q = -1 - n/\sum_i^n \log X_i$

5. Flow time of certain product has been measured in a workshop for ten selected pieces. The resulting values were (in minutes): 17, 21, 14, 23, 20, 24, 19, 19, 25 and 18. It is assumed that the flow time of the studied product is normally distributed. Determine point estimators of mean and standard deviation of the flow time. R: $\mu = 20\,\mathrm{min}$, $\sigma = 3.37\,\mathrm{min}$