Additional problems for exercises in Random phenomena course - 4th set

1. A random variable X has the probability density function $f_X(x) = x/8$ for $0 \leq x < 4$. Determine the probability density function of a variable $Y = 2X + 4$. R: $f_Y(y) = (y-4)/32$

2. A random variable X has the probability density function:

$$f_X(x) = \begin{cases} \dfrac{C}{1+(x-1)^2}\,, & \text{for } 0 \leq x \leq 2\,, \\ 0\,, & \text{for } x \text{ elsewhere}\,. \end{cases}$$

   Determine the probability density function of variable $Y = (X-1)^2$. R: $f_Y(y) = 2/[\pi\sqrt{y}(1+y)]$

3. Lifetime of a machine component is exponentially distributed with an average of $1/\lambda$. When a component fails, it is replaced with a spare, which has the same characteristics as the original component and both are independent of each other. The machine works as long as one of the two components work. Determine the probability density function of the machine lifetime. R: $f_Z(z) = \lambda^2 z\,\mathrm{e}^{-\lambda z}$

4. The telephone office monitors length of calls. $40\,\%$ of calls last a minute, $30\,\%$ of calls last two minutes, $20\,\%$ of calls last three minutes and $10\,\%$ of calls last four minutes. What are the mean and variance of the call length? R: $\mathrm{E}\,[X] = 2\,\mathrm{min}$, $\mathrm{Var}\,[X] = 1\,\mathrm{min}^2$

5. Determine mean and variance of number of dots in throwing die. R:$\mathrm{E}\,[X] = 7/2$, $\mathrm{Var}\,[X] = 35/12$

6. Determine mean and variance of Poisson random variable. R: $\mathrm{E}\,[X] = \lambda$, $\mathrm{Var}\,[X] = \lambda$

7. Determine mean and variance of uniformly distributed random variable in the interval $[a, b]$ R: $\mathrm{E}\,[X] = (a+b)/2$, $\mathrm{Var}\,[X] = (a-b)^2/12$

8. Determine mean and variance of exponentially distributed random variable. R: $\mathrm{E}\,[X] = 1/\lambda$, $\mathrm{Var}\,[X] = 1/\lambda^2$