Problems for exercises in Random phenomena course - 4th set

1. Random variable X has a probability distribution $f_X(x) = 2x/9$ for $0 \leq x < 3$. Find the probability distribution of the random variable $Y = 2X + 5$. R: $f_Y(y) = (y - 5)/18$

2. Lifetime of two light bulbs is exponentially distributed with averages of $1/a$ and $1/b$, respectively, where $a \neq b$ and $a, b \neq 0$. Light bulbs are connected in such a way that only one is in use. When the first one burns out the second lights up. Determine the probability density function of the total time of lighting. R: $f_Z(z) = ab(a - b)^{-1} \left(\mathrm{e}^{-bz} - \mathrm{e}^{-az}\right)$

3. A worker tends to N^2 machines. Due to small failures he must intervene now at this and then on another machine. Since the machines are identical it is assumed that the probability of failure is the same for all machines. Machines are placed in m rows by n machines so that the distance between the two machines in the adjacent column or adjacent row is equal to a. The worker reaches the machine with failure by the shortest route, walking around the (square-footprint) machines. In how many rows should the machines be placed to ensure the shortest average route for the worker? R: N rows by N machines, $\mathrm{E}[s] = 2a(N^2 - 1)/(3N)$

4. A player throws the dice. If the result is an even number, the player gets the same amount of money units. If the result is an odd number, the player pays the same amount of money units. Calculate the number of money units the palyer can expect to have after 100 throws. R: 50

5. A player throws two dices and gets a number of money units equal to the product of both dice results. How many money units can he expect to have after 100 throws? R: 1225

6. Determine mean and variance of the binomial random variable. R:$\mathrm{E}[X] = np$, $\mathrm{Var}[X] = np(1 - p)$

7. Determine mean and variance of the standard normal random variable. R: $\mathrm{E}[X] = 0$, $\mathrm{Var}[X] = 1$