Functions of random variables: Suppose that X is a continuous random variable with probability distribution $f_X(x)$ and that the random variable Y is defined by a function $Y = g(X)$. Then, the probability distribution $f_Y(y)$ can be calculated by using the inverse function $X = h(Y) = g^{-1}(Y)$:

$$f_Y(y) = f_X(h(y)) \left| \frac{\mathrm{d}h(y)}{\mathrm{d}y} \right| . \tag{1}$$

The above equation is valid for a monotonic function $g(X)$. If $g(X)$ is not monotonic, it should be divided on k piecewise monotonic parts $g_i(X)$ with the corresponding inverses $h_i(Y)$:

$$f_Y(y) = \sum_{i=1}^{k} f_X(h_i(y)) \left| \frac{\mathrm{d}h_i(y)}{\mathrm{d}y} \right| . \tag{2}$$

Scalar function of vector random variable: Suppose that X and Y are two continuous random variables with joint probability distribution $f_{XY}(x, y)$ and that random variable Z is defined by $Z = g(X, Y)$. Then, the calculation of the probability distribution $f_Z(z)$ in general depends on $g(X, Y)$. In the most simple case with $Z = X + Y$ the calculation is:

$$f_Z(z) = \int_{-\infty}^{\infty} f_{XY}(x, z - x)\ \mathrm{d}x \overset{\text{(if } X \text{ and } Y \text{ independent)}}{=} \int_{-\infty}^{\infty} f_X(x) f_Y(z - x)\ \mathrm{d}x\,. \tag{3}$$

Raw moment of the k-th order is defined for discrete and continuous random variables by:

$$\begin{aligned} m_k &= \mathrm{E}\left[X^k\right] = \sum_{i=1}^{n} x_i^k P\left(X = x_i\right)\ , \\ m_k &= \mathrm{E}\left[X^k\right] = \int_{-\infty}^{\infty} x^k f(x)\,\mathrm{d}x\,. \end{aligned} \tag{4}$$

Central moment of the k-th order is defined for discrete and continuous random variables by:

$$\begin{aligned} \mu_k &= \mathrm{E}\left[(X - \mathrm{E}[X])^k\right] = \sum_{i=1}^{n} (x_i - \mathrm{E}[X])^k P\left(X = x_i\right)\ , \\ \mu_k &= \mathrm{E}\left[(X - \mathrm{E}[X])^k\right] = \int_{-\infty}^{\infty} (x - \mathrm{E}[X])^k f(x)\,\mathrm{d}x\,. \end{aligned} \tag{5}$$

The raw moment of the first order m_1 is named **mean** m, the central moment of the second order μ_2 is named **variance** $\mathrm{Var}[X]$. The variance can also be expressed by:

$$\mathrm{Var}[X] = \mathrm{E}\left[X^2\right] - (\mathrm{E}[X])^2\ . \tag{6}$$

Raw and central moments of a vector random variable are defined in similar way. For a two-dimensional continuous vector random variable the definitions are:

$$\begin{aligned} \mathrm{E}\left[X^j Y^k\right] &= \iint x^j y^k f_{XY}(x, y)\,\mathrm{d}x\,\mathrm{d}y\,, \\ \mathrm{E}\left[(X - \mathrm{E}[X])^j (Y - \mathrm{E}[Y])^k\right] &= \iint (x - \mathrm{E}[X])^j (y - \mathrm{E}[Y])^k f_{XY}(x, y)\,\mathrm{d}x\,\mathrm{d}y\,. \end{aligned} \tag{7}$$

Mostly used are the first raw moment $\mathrm{E}[XY]$ named **correlation** $\mathrm{Cor}[X, Y]$ and the first central moment $\mathrm{E}[(X - m_X)(Y - m_Y)]$ named covariance $\mathrm{Cov}[X, Y]$. They are related by:

$$\mathrm{Cov}[X, Y] = \mathrm{Cor}[X, Y] - m_X m_Y\,. \tag{8}$$