Additional problems for exercises in Random phenomena course - 3rd set

1. For a machine, time before failure is exponentially distributed with an average of 10^4 h.
   (a) What is the probability that the machine fails in the first 1000 h of operation? R: P =0.095
   (b) What is the probability that the machine will operate without failure at least 10000 h? R: P =0.367
   (c) The machine operates without failure for 5000 h. What is the probability that it will break down in the following 1000 h? R: $P = 0.095$

2. Weld length made with one electrode of certain type is normally distributed with average of 1 m and standard deviation of 15 cm.
   (a) What is the probability that a weld not longer than 0.8 m is made with one electrode?R: $P = 0.091$
   (b) Which one-electrode weld length is exceeded in 99 % of cases? R: $l = 0.65$ m

3. Some airlines grant more reservations for a plane than there are available seats because they want to have full occupancy of the aircraft. Therefore, for an aircraft with 100 seats they grant reservations for 150 passengers. The probability that a passenger who has a reservation does not appear at the airport is 0.4. Passengers behave independently of each other.
   (a) What is the probability that at least one passenger with a reservation is left without a seat on the plane? R: $P = 0.039$ and $P = 0.040$
   (b) What maximum number of reservations can be granted to have too many passengers in not more than 1 % of cases? R: $C = 144$ and $C = 144$.

4. In assembly of the product a piece having a hole has to be joined with a piece having a plug. Depth of the hole and height of the plug are normally distributed. Average depth of the hole is 10.50 mm with a standard deviation of 0.20 mm, while the plug has an average height of 10.38 mm with a standard deviation of 0.18 mm.
   (a) What is the probability that a randomly selected plug is too high for a randomly selected hole? R: $P = 0.328$
   (b) What should the average depth of hole at a given standard deviation and at given plug distribution be to have the probability of the first problem case less than 0.05? R: $m_I > 10.82$ mm

5. Daily consumption of water in the village is limited to 1000 m^3. Daily water consumption of households is normally distributed with an average of 600 m^3 and standard deviation of 70 m^3. Daily water consumption of industry is also normally distributed with an average of 200 m^3 and a standard deviation of 40 m^3. In addition to the household and industry consumption, water is also used for other purposes. What is the maximal amount of water that can be daily used for other purposes so that the village limit consumption is not exceeded with a probability of at least 0.95? R: $V < 67.4\,m^3$

6. We have 10000 m^3 of gas in stock. It is assumed that the daily gas consumption is normally distributed with an average of 50 m^3 and a standard deviation of 5 m^3. It is also assumed that the consumption of a randomly selected day is independent of the consumption in other days. For how many days will our gas stock last, if there has to be enough gas in every day with a probability of at least 0.95? R: $n = 197$

Note: To solve some of the problems a tabulated Gaussian probability distribution is required (Table A.1 in the textbook *Opis naključnih pojavov*).