

A **probability density function** f_X of a continuous random variable X is such that:

$$\begin{aligned} f_X(x) &\geq 0, \\ \int_{-\infty}^{\infty} f_X(x) dx &= 1, \\ P(a \leq X \leq b) &= \int_a^b f_X(x) dx. \end{aligned} \quad (1)$$

The corresponding **cumulative distribution function** F_X of a continuous random variable X is:

$$F_X(x) = P(X \leq x) = \int_{-\infty}^x f_X(u) du \quad \text{for} \quad -\infty < x < \infty. \quad (2)$$

Given F_X , the f_X can be calculated by:

$$\frac{d}{dx} F_X(x) = \frac{d}{dx} \int_{-\infty}^x f_X(u) du = f_X(x). \quad (3)$$

A **continuous uniform random variable** X over the interval $[a, b]$ has the following probability density function $f(x)$ and cumulative distribution function $F(x)$:

$$f(x) = \frac{1}{b-a}, \quad F(x) = \int_a^x f(u) du = \frac{x-a}{b-a}, \quad \text{for} \quad a \leq x \leq b. \quad (4)$$

An **exponential random variable** X with the mean $1/\lambda > 0$ has the following probability density function $f(x)$ and cumulative distribution function $F(x)$:

$$f(x) = \lambda e^{-\lambda x}, \quad F(x) = \int_0^x f(u) du = 1 - e^{-\lambda x}, \quad \text{for} \quad x \geq 0. \quad (5)$$

A **normal (Gauss) random variable** X with the mean $m \in \mathbb{R}$ and the standard deviation $\sigma > 0$ has the following probability density function $f(x)$ and cumulative distribution function $F(x)$:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}}, \quad F(x) = \int_{-\infty}^x f(u) du = 0.5 + \Phi\left(\frac{x-m}{\sigma}\right), \quad \text{for} \quad x \in \mathbb{R}. \quad (6)$$

Here, $\Phi\left(\frac{x-m}{\sigma}\right) = \Phi(z)$ is the Laplace function:

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{u^2}{2}} du, \quad (7)$$

which is tabulated for different values of z in the table A.1 of the textbook *Opis naključnih pojavov*. The Laplace function has the following properties:

$$\Phi(\infty) = 0.5 \quad \text{and} \quad \Phi(-z) = -\Phi(z). \quad (8)$$

In Equation (6) a **standard normal random variable** Z has been introduced:

$$Z = \frac{X - m}{\sigma}. \quad (9)$$

The probability density function of a normal random variable X is usually shortly denoted by $\mathcal{N}(x; m, \sigma)$. The probability density function of a standard normal random variable Z is $\mathcal{N}(z; 0, 1)$ and its probabilities can be calculated using the table A.1 of the textbook *Opis naključnih pojavov*. The transformation (9) is referred to as *standardizing*.

When the parameter n of a binomial distribution is large and the probability of a success is $p \approx 0.5$, the binomial distribution can be approximated by a normal distribution by using:

$$m = np \quad \text{and} \quad \sigma = \sqrt{np(1-p)}. \quad (10)$$

By an alternative criteria the approximation is good when $np > 5$ and $n(1-p) > 5$.

The normal distribution can be used as an approximation of the Poisson distribution when $\lambda > 5$. The parameters of a normal distribution are then:

$$m = \lambda \quad \text{and} \quad \sigma = \sqrt{\lambda}. \quad (11)$$

Addition (subtraction) of two independent normal random variables X_1 and X_2 with the probability density functions $\mathcal{N}(x_1; m_1, \sigma_1)$ and $\mathcal{N}(x_2; m_2, \sigma_2)$ results in a normal random variable $Y = X_1 \pm X_2$ with the probability density of:

$$\mathcal{N}(y; m_y, \sigma_y), \quad \text{where} \quad m_y = m_1 \pm m_2 \quad \text{and} \quad \sigma_y = \sqrt{\sigma_1^2 + \sigma_2^2}. \quad (12)$$