Probability mass function f_X of a discrete random variable X with possible values $\{x_i\}$ is defined by:

$$\begin{aligned} f_X(x_i) &= P\left(X = x_i\right) = p(x_i), \\ f_X(x_i) &\geq 0, \\ \sum_{x_i \in S_X} f_X(x_i) &= 1. \end{aligned} \tag{1}$$

It satisfies also the following property:

$$P\left(a \leq X \leq b\right) = \sum_{x_i \in [a,b]} f_X(x_i). \tag{2}$$

The cumulative distribution function F_X of a discrete random variable X is defined by:

$$F_X(x_i) = P\left(X \leq x_i\right) = \sum_{x_j \leq x_i} f_X(x_j). \tag{3}$$

The empirical mean value of a discrete random variable X, determined from N repetitions of the random experiment:

$$\langle X \rangle = \sum_k x_k \, \frac{N_k}{N}. \tag{4}$$

The statistical mean of a discrete random variable X:

$$\mathrm{E}\left[X\right] = \sum_k x_k \; f_X(x_k). \tag{5}$$

The variance of a discrete random variable X:

$$\sigma^2 = \mathrm{Var}\left[X\right] = \mathrm{E}\left[\left(X - \mathrm{E}\left[X\right]\right)^2\right] = \sum_k (x_k - \mathrm{E}\left[X\right])^2 \; f_X(x_k) = \mathrm{E}\left[X^2\right] - \mathrm{E}\left[X\right]^2 . \tag{6}$$

Bernoulli random experiment results in only two possible results (eg success and failure). If the n trials of the random experiment are independent and if the probability of a success p in each trial is constant then the number of trials resulting in a success X is a binomial random variable and its probability mass function f_X is a binomial distribution:

$$f_X(x) = P\left(X = x\right) = \binom{n}{x} p^x \left(1-p\right)^{n-x} , \qquad x = 0, 1, \ldots, n \,. \tag{7}$$

The binomial coefficient is defined by:

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \,. \tag{8}$$

In case the number of trials n is large and the probability of a success p is low, so that $np \sim 1$, the probability mass function of the binomial distribution can be approximated by the probability mass function of the Poisson distribution where the parameter $\lambda = np$ is used:

$$f_X(x) = P\left(X = x\right) = \mathrm{e}^{-\lambda} \, \frac{\lambda^x}{x!} \,, \qquad x = 0, 1, \ldots \,. \tag{9}$$

The parameter λ equals the statistical mean of the binomial/Poisson random variable X. In the Poisson distribution, λ can be interpreted as a product of a mean frequency of successful results ν and the duration of the trial, that can be either time t, length l etc.:

$$\lambda = \nu \cdot t \qquad \text{or} \qquad \lambda = \nu \cdot l \qquad \text{etc.} \tag{10}$$