Events

Definitions: S = sample space/certain event; $\emptyset$ = impossible event; $A, B, C, ... \subset S$ = events.

	intersection ($A \cap B$)	union ($A \cup B$)
Commutativity:	$A \cap B = B \cap A$	$A \cup B = B \cup A$
Associativity:	$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$
Subset:	$(A \cap B) \subset A, (A \cap B) \subset B$	$A \subset (A \cup B), B \subset (A \cup B)$
Misc.:	$A \subset B \Rightarrow A \cap B = A$	$A \cup \emptyset = A, A \cup A = A$
Distributivity:	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Complement (A^C):

$$\begin{aligned} (A^C)^C = A, &\quad A \subset B \Rightarrow B^C \subset A^C \\ A \cap A^C = \emptyset, &\quad A \cup A^C = S \end{aligned} \tag{1}$$

DeMorgan's laws:

$$(A \cup B)^C = A^C \cap B^C, \quad (A \cap B)^C = A^C \cup B^C \tag{2}$$

Interesting: $(\cup, \cap, \subset, \supset, S, \emptyset) \overset{C}{\Longleftrightarrow} (\cap, \cup, \supset, \subset, \emptyset, S)$

Probability

Probability of a complementary event:

$$P\left(A^C\right) = 1 - P(A) \tag{3}$$

Probability of the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{4}$$

Probability of the intersection of <u>mutually exclusive</u> events:

$$A \cap B = \emptyset \quad \Rightarrow \quad P(A \cap B) = 0 \tag{5}$$

Conditional probability of event A given the event B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad \text{where} \quad P(B) > 0 \tag{6}$$

- commutativity of intersection: $P(A \cap B) = P(A|B)\,P(B) = P(B \cap A) = P(B|A)\,P(A)$
- A and B are <u>independent</u> events $\Leftrightarrow P(A|B) = P(A) \Leftrightarrow P(B|A) = P(B) \Leftrightarrow P(A \cap B) = P(A) \cdot P(B)$ (7)

Mutually exclusive and exhaustive set $\{A_1, A_2, ..., A_n\}$:

$$\bigcup_{i=1}^{n} A_i = S, \quad A_i \cap A_j = \emptyset \quad \text{for } i \neq j \tag{8}$$

Bayes theorem ($\{A_i\}$ is mutually exclusive and exhausting set):

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(A_j)\,P(B|A_j)}{\sum_{i=1}^{n} P(A_i)\,P(B|A_i)} \tag{9}$$