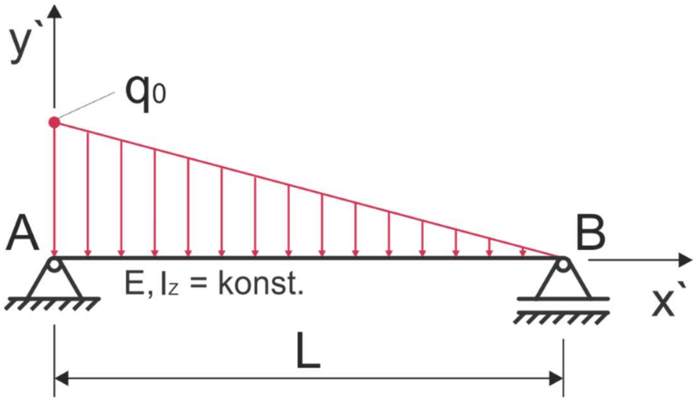
**Dodatna naloga 9**

1.) Določite enačbo upogibnice ter mesto in velikost največjega povesa za nosilec na sliki.

$q_0, L, E, I_z$ – vzemite kot znane vrednosti

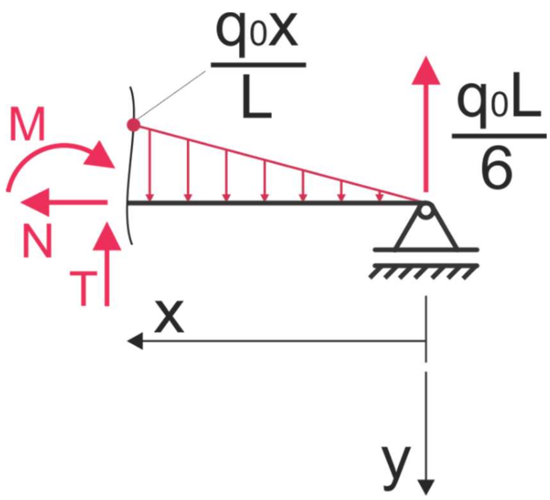
$y(x) = ?,\ x_{\mathrm{MAX}} = ?,\ y_{\mathrm{MAX}} = ?$




Rešitev:

- izračunamo reakcijske sile:

$$A_{x`} = 0,\ \ A_{y`} = \frac{q_0 L}{3},\ \ B = \frac{q_0 L}{6}$$




- izračunamo notranji upogibni moment:

$$M(x) = -\frac{q_0 x^3}{6L} + \frac{q_0 L x}{6}$$

- enačbo momenta vstavimo v diferencialno enačbo upogibnice in dvakrat integriramo:

$$y``(x) = -\frac{M(x)}{EI_z} = \frac{q_0}{EI_z}\left(\frac{x^3}{6L} - \frac{Lx}{6}\right)$$

$$y`(x) = \frac{q_0}{EI_z}\left(\frac{x^4}{24L} - \frac{Lx^2}{12}\right) + C_1$$

$$y(x) = \frac{q_0}{EI_z}\left(\frac{x^5}{120L} - \frac{Lx^3}{36}\right) + C_1 x + C_2$$

- zapišemo robne pogoje in izračunamo vrednost integracijskih konstant:

$$y(x=0) = 0 \quad \Rightarrow \quad C_2 = 0$$

$$y(x=L) = 0 \quad \Rightarrow \quad C_1 = \frac{7 q_0 L^3}{360 EI_z}$$

- poiščemo maksimum funkcije upogibnice. Ta bo zagotovo v stacionarni točki funkcije, saj na robovih polja ne more biti. Namig: uvedba nove spremenljivke $a = x^2/L^2$ za izračun stacionarnih točk.

$$y`(x) = 0 \quad \Rightarrow \quad x_{MAX} = \sqrt{1 - \frac{2\sqrt{30}}{15}}\,L \quad \text{(dobimo tudi še tri stacionarne točke izven polja)}$$

$$y_{MAX} \approx 0{,}0065222 \frac{q_0 L^4}{EI_z}$$

2.) Določite največji poves nosilca in zasuk nosilca v členkastih podporah.

$F = 3\ \text{kN}$

$q = 4\ \text{kN/m}$

$L = 1\ \text{m}$

$E = 200000\ \text{MPa}$

$I_z = 5 \cdot 10^5\ \text{mm}^4$

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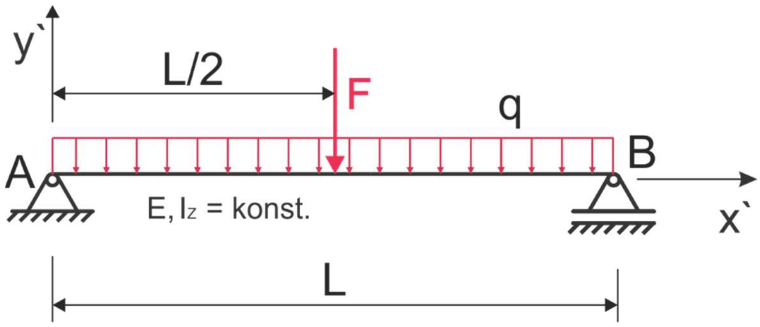
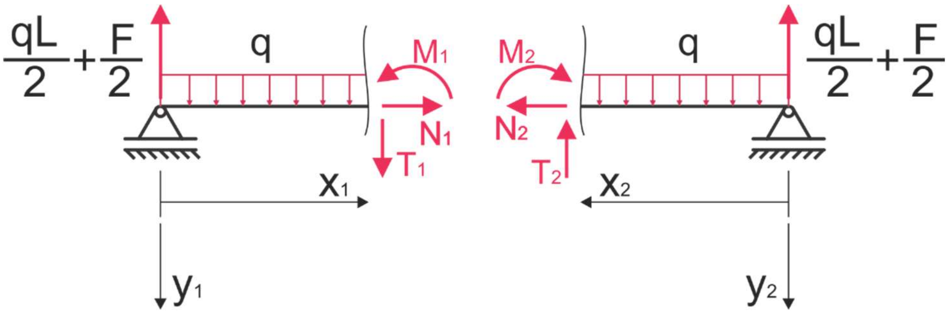
$y(x) = ?,\ x_{\text{MAX}} = ?,\ y_{\text{MAX}} = ?$




Rešitev:

- izračunamo reakcijske sile:

$$A_{x`} = 0,\ \ A_{y`} = \frac{qL}{2} + \frac{F}{2},\ \ B = \frac{qL}{2} + \frac{F}{2}$$




- izračunamo notranji upogibni moment:

$$M_1(x_1) = \frac{1}{2}(F + qL)x_1 - \frac{q{x_1}^2}{2},\quad M_2(x_2) = \frac{1}{2}(F + qL)x_2 - \frac{q{x_2}^2}{2}$$

- enačbo momenta vstavimo v diferencialno enačbo upogibnice in dvakrat integriramo. Dobimo:

$$y_1(x_1) = \frac{1}{2EI_z}\left(\frac{q{x_1}^4}{12} - (F + qL)\frac{{x_1}^3}{6}\right) + C_1x_1 + C_2,\ \ y_2(x_2) = \frac{1}{2EI_z}\left(\frac{q{x_2}^4}{12} - (F + qL)\frac{{x_2}^3}{6}\right) + C_3x_2 + C_4$$

- zapišemo robne pogoje in izračunamo vrednost integracijskih konstant:

$$y_1(x_1 = 0) = 0,\ \ y_2(x_2 = 0) = 0,\ \ y_1(x_1 = L/2) = y_2(x_2 = L/2),\ \ y`_1(x_1 = L/2) = -y`_2(x_2 = L/2)$$

$$C_2 = C_4 = 0,\ \ C_1 = C_3 = \frac{1}{EI_z}\left(\frac{qL^3}{24} + \frac{FL^2}{16}\right)$$

- poiščemo maksimalni poves nosilca (preverimo robove polj in stacionarne točke). Dobimo rezultat:

$$y_{\text{MAX}} = y_1(x_1 = L/2) = y_2(x_2 = L/2) = \frac{5qL^4}{384EI_z} + \frac{FL^3}{48EI_z}$$

- izračunamo še zasuk nosilca v podporah: $\alpha \approx y`_1(x_1 = 0) = y`_2(x_2 = 0) = C_1 = \frac{17}{4800}\ \text{rad}$

**Opomba**: zaradi simetrije se da nalogo rešiti z zapisom ene same upogibnice in robnimi pogoji:

$$y_1(x_1 = 0) = 0,\quad y`_1(x_1 = L/2) = 0$$

3.) Izračunajte vertikalni poves na sredini nosilca.

$q = 6$ kN/m

$L = 1$ m

$E = 200000$ MPa

$I_z = 5 \cdot 10^5$ mm$^4$

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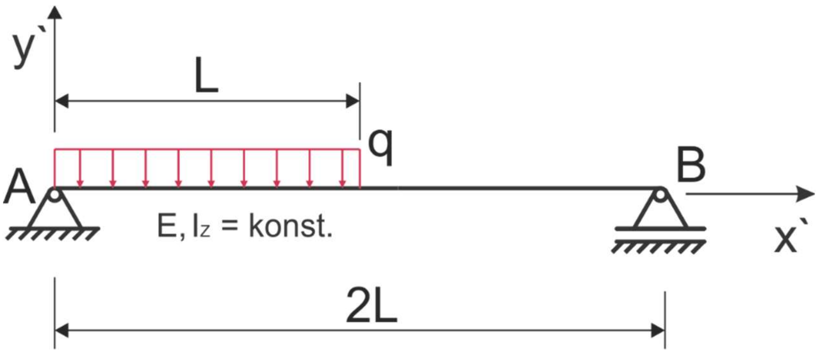
$y(x` = L) = ?$




Rezultat:

$$y(x` = L) = \frac{5qL^4}{48EI_z} = 6{,}25 \text{ mm}$$

4.) Določite $M_0$ tako, da bo:

a) poves na koncu nosilca enak nič

b) poves na sredini nosilca enak nič

$q, L, E, I_z$ – vzemite kot znane vrednosti

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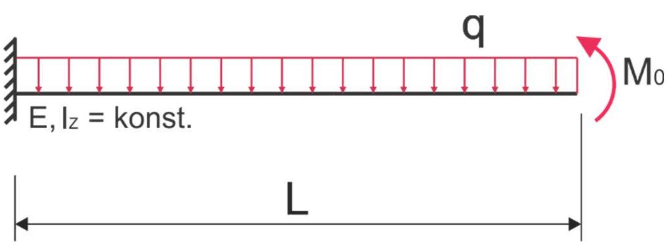
$M_0 = ?$

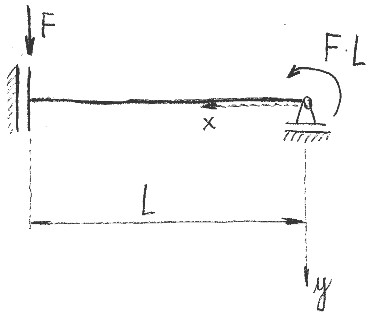



Rezultat:

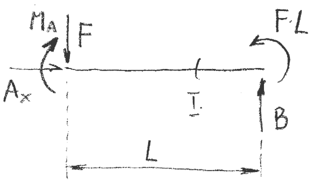
a) $M_0 = \frac{qL^2}{4}$

b) $M_0 = \frac{17qL^2}{48}$

Določite mesto in velikost največjega povesa nosilca.




reakcije:




$A_x = 0$

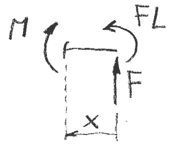
$B - F = 0$

$F \cdot L + F \cdot L - M_A = 0$

$B = F$

$M_A = 2FL$

I polje




$0 \leq x \leq L$

$M - Fx - F \cdot L = 0$

$M = F \cdot (L + x)$

$$y'' = -\frac{M}{EJ_z} = -\frac{F(L+x)}{EJ} = -\frac{F}{EJ} \cdot (L+x)$$

$$y' = -\frac{F}{EJ_z}\left(Lx + \frac{x^2}{2}\right) + C_1$$

$$y = -\frac{F}{EJ_z}\left(\frac{Lx^2}{2} + \frac{x^3}{6}\right) + C_1 \cdot x + C_2$$

R.P.:

$y(0) = 0 \Rightarrow C_2 = 0$

$y'(L) = 0 \Rightarrow C_1 = \frac{3FL^2}{2EJ_z}$

$$y = -\frac{F}{EJ_z} \cdot \left(\frac{x^3}{6} + \frac{Lx^2}{2}\right) + \frac{3FL^2 x}{2EJ_z}$$

$$y = -\frac{F}{EJ_z}\left(\frac{x^3}{6} + \frac{Lx^2}{2} - \frac{3L^2 x}{2}\right)$$

pogoj za največji poves: $y'(x) = 0$

$$-\frac{F}{EJ_z} \cdot \left(Lx + \frac{x^2}{2}\right) + \frac{3FL^2}{2EJ_z} = 0$$

$$-Lx - \frac{x^2}{2} + \frac{3L^2}{2} = 0 \quad \{ \cdot 2$$

$$-2Lx - x^2 + 3L^2 = 0 \quad \{ \cdot (-1)$$

$$x^2 + 2Lx - 3L^2 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2L \pm \sqrt{4L^2 - 4 \cdot 1 \cdot (-3L^2)}}{2}$$

$$x_{1,2} = \frac{-2L \pm 4L}{2} = -L \pm 2L$$

$x_1 = L$ ✓

$x_2 = -3L$ ✗

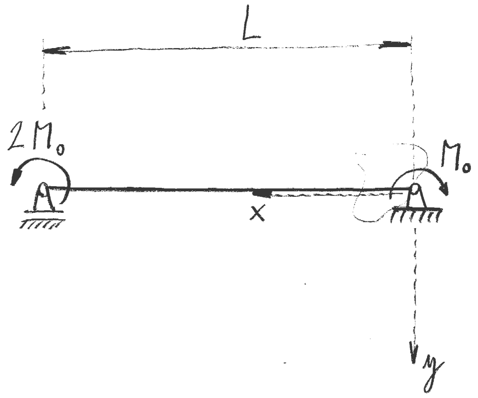
največji poves je pri $x = L$:

$$y(L) = -\frac{F}{EJ_z} \cdot \left(\frac{L^3}{6} + \frac{L^3}{2} - \frac{3L^3}{2}\right)$$

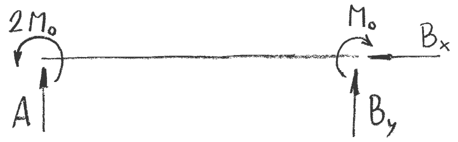
$$y(L) = -\frac{F}{EJ_z} \cdot \frac{L^3 + 3L^3 - 9L^3}{6} = \frac{5FL^3}{6EJ_z}$$

$$y_{MAX} = y(L) = \frac{5FL^3}{6EJ_z}$$

Določite mesto in velikost največjega povesa.




reakcije:

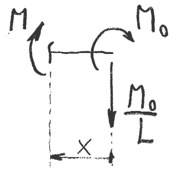



$B_x = 0$

$A + B_y = 0$

$2M_0 - M_0 + B_y \cdot L = 0$

$B_y = -\frac{M_0}{L} \Rightarrow A = \frac{M_0}{L}$




$M + M_0 + \frac{M_0}{L}x = 0$

$M = -M_0 - \frac{M_0}{L}x$

$0 \leq x \leq L$

$y'' = -\frac{M}{EJ_z} = \frac{M_0 + \frac{M_0}{L}x}{EJ_z}$

$y' = \frac{M_0}{EJ_z}x + \frac{M_0}{LEJ_z}\frac{x^2}{2} + C_1$

$y = \frac{M_0}{EJ_z}\frac{x^2}{2} + \frac{M_0}{LEJ_z}\cdot\frac{x^3}{6} + C_1 \cdot x + C_2$

$y(0) = 0 \Rightarrow C_2 = 0$

$y(L) = 0$

$\frac{M_0}{EJ_z}\frac{L^2}{2} + \frac{M_0}{LEJ_z}\frac{L^3}{6} + C_1 \cdot L = 0$

$\frac{M_0 L^2}{2EJ_z} + \frac{M_0 L^2}{6EJ_z} = -C_1 L$

$C_1 = -\frac{M_0 L}{2EJ_z} - \frac{M_0 L}{6EJ_z}$

$C_1 = -\frac{2M_0 L}{3EJ_z}$

$y'(x) = 0$

$\frac{M_0}{EJ_z}x + \frac{M_0}{LEJ_z}\cdot\frac{x^2}{2} - \frac{2M_0 L}{3EJ_z} = 0$

$x + \frac{x^2}{2L} - \frac{2L}{3} = 0$

$2Lx + x^2 - \frac{4L^2}{3} = 0$

$6Lx + 3x^2 - 4L^2 = 0$

$3x^2 + 6Lx - 4L^2 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6L \pm \sqrt{36L^2 + 48L^2}}{6}$

$x_{1,2} = \frac{-6L \pm \sqrt{84} \cdot L}{6} = -L \pm 1{,}5275 \cdot L = -L \pm \sqrt{\frac{21}{9}} L$

$x_{MAX} = 0{,}5275 \cdot L = \left(\sqrt{\frac{21}{9}} - 1\right) \cdot L$

$y_{MAX} \approx -0{,}1881 \cdot \frac{M_0 L^2}{EJ_z}$

~~$x_{MAX} =$~~

$x_{MAX} = \left(\sqrt{\frac{7}{3}} - 1\right) \cdot L$