Experimental Identification of the Dynamic Piezoresistivity of Fused-Filament-Fabricated Structures

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Cite as:

Abstract

Fused-filament-fabrication (FFF) technology is promising for the production of fully embedded piezoresistive dynamic sensors. However, the lack of experimental identification of the dynamic piezoresistivity limits the scientific and applied progress. Dynamic piezoresistivity of 3D printed structures is hard to research due to: structural anisotropy and heterogeneity, the large number of process parameters in 3D printing, the nonhomogenous electric/stress/strain fields. Additionally, the piezoresistivity can be dependent on the frequency of the mechanical load and also on the temperature. This research proposes an experimental method to identify the dynamic piezoresistivity of unidirectionally printed specimens. The method is based on the Bridgman model of piezoresistivity and is extended to the dynamic conditions. With a single measurement, the stress in the specimen, the initial resistivity and the piezoresistive coefficient in the frequency domain are identified. The applicability of the method is experimentally tested on three specimens with different orientations of the mechanical and electrical loads. The identified piezoresistive coefficients identified using the proposed method can be used in analytical and numerical models in embedded, FFF dynamic sensors and similar applications.

Keywords: dynamic piezoresistivity, fused filament fabrication, piezoresistive coefficient, electric resistivity, frequency domain method, additive manufacturing

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1. Introduction

In recent years the production of smart materials suitable for additive manufacturing technologies has enabled the simultaneous production of multifunctional devices \[1\] \emph{e.g.}, actuators \[2, 3\], embedded electric circuits \[4, 5\] and sensors \[6, 7\]. In the case of fused-filament-fabrication (FFF) technology, piezoresistive materials can be used to create sensors. Piezoresistivity denotes the phenomenon, whereby the electrical resistivity changes under the influence of mechanical strain \[8\].

In 2012 Leigh \textit{et al.} developed a conductive thermoplastic material called “carbomorh” and showed that it is possible to use it with a 3D printer to print parts able to sense strain \[9\]. Recent years have seen a significant increase in the number of 3D printed sensors, \emph{e.g.}, Alsharari \textit{et al.} \[10\] researched graphene-based polylactic acid (PLA) and thermoplastic polyurethane (TPU) composite-based sensors, Christ \textit{et al.} \[11\] researched TPU/multiwalled carbon nanotube (TPU/MWCNT) composite-based sensors, Goeding \textit{et al.} \[12\] researched embedded tensile-strain sensors, Kim \textit{et al.} researched a multiaxial force sensor \[13\], and Al-Rubaiai \textit{et al.} researched a strain sensor for sensing wind \[14\].

In 2019 Maurizi \textit{et al.} researched dynamic strain measurements of FFF structures \[15\] and showed that it might be possible in the future to use FFF technology to produce reliable dynamic sensors. However, the manufacturing of reliable dynamic sensors is currently limited, since the influence on the dynamic piezoresistivity of FFF structures with applied loads in different directions is not well researched.

The FFF structure has to be electrically conductive in order to exhibit piezoresistive properties. Most of the materials used in the FFF process are, however, thermoplastics, which are electrically non-conductive. Conductivity is achieved by dispersing conductive particles in a non-conductive matrix: the conductivity of polymer composites is highly dependent on the volume ratio of the conductive particles, which is explained by the percolation theory \[16\]. Other mechanisms affecting the conductivity are tunneling and temperature effects \[17\]. In recent years the volume ratio of conductive parts was often researched, \emph{e.g.}, composites of ABS/carbon nanotubes (CNT) \[18\], ABS/MWCNT \[19\], nylon 6/metal and high-density polyethylene \[20\], polypropylene/carbon black \[21\] and TPU/MWCNT \[11\]. Also investigated were the influences of the FFF process parameters on the electrical resistivity, \emph{e.g.}, Tan \textit{et al.} \[20\] researched the influence of the printing
orientation, Hampel et al. [22] the layer height, nozzle temperature and printing velocity, Zhang et al. [23] the layer thickness, raster width, and air-gap, and Watschke et al. researched the orientation, velocity and material flow [24]. In these studies a lower resistivity along the direction of the deposited material was achieved in comparison to the transverse direction. Furthermore, a 30-times-higher resistivity can be identified when poor electrical contacts are applied to the specimen [24]. The applicability of FFF as manufacturing technology for electric circuits and components was studied by Flowers et al. [25].

On the other hand, piezoresistivity as a material property has rarely been studied directly. While the piezoresistive behaviour of inkjet-printed thin films [26] and isotropic nanocomposites [27] has already been examined in terms of the tensor-based resistivity-strain relationship, the evaluation of the piezoresistivity of the FFF structures was limited to the resistance-change observations and gauge factors. For static and quasistatic FFF sensors, the influence of the geometry of the sensing part was researched by Gooding et al. [12], while Christ et al. [11] examined the influence of the content of conductive particles in a non-conductive polymer and Dawoud et al. [28] investigated the influence of raster gap and raster orientation on the static piezoresistivity. Dawoud et al. applied a static tensile load to a specimen and used a two-probe resistance measurement with different configuration electrodes; depending on the configuration of the electrodes, different piezoresistive properties were obtained, which indicates that electric boundary effects have an important impact on the piezoresistivity.

An important factor relating to the absence of dynamic piezoresistivity research is the lack of methods for dynamic piezoresistivity identification. Several difficulties arise when the piezoresistivity of FFF structures is identified e.g., the anisotropy, the heterogeneity, the large number of 3D printing process parameters, the difficulty with electrical contacts, the dependency of the load frequency and finally the temperature dependence. Due to the anisotropy and heterogeneity it is difficult to simultaneously establish a homogenous electric and stress/strain field. To overcome the existing issues, a new method is proposed, based on unidirectionally printed specimens that can be used to estimate the dynamic piezoresistivity of FFF structures with the mechanical load and electrical field established in the same direction. Since many of the design principles of piezoresistive dynamic sensors are based on the electric field and stress being parallel [8], the obtained coefficients are widely applicable. Coefficients can be used in numerical and analytical models and to estimate the effects of process parameters.

This manuscript is organized as follows: Sec. 2 gives the theoretical
background on the theory of elasticity and electric conductivity, Sec. 3 introduces the method for dynamic piezoresistivity identification, Sec. 4 gives the background on the experimental research, Sec. 5 presents the experimental results and, finally, Sec. 7 gives the conclusions.

2. Theoretical background

The large variety of possible raster patterns results in the local microstructural behavior of FFF structures. However, effective homogenized properties can be used to predict their macroscopical behaviour [29, 30], especially when the infill density is close to 100%. Ohm’s law for a homogeneous anisotropic structure relates the electric field intensity \( E \) and the electric current density \( J \) through the resistivity matrix \( \rho \) as [31]:

\[
\begin{bmatrix}
  E_1 \\
  E_2 \\
  E_3 
\end{bmatrix} =
\begin{bmatrix}
  \rho_{11} & \rho_{12} & \rho_{13} \\
  \rho_{21} & \rho_{22} & \rho_{23} \\
  \rho_{31} & \rho_{32} & \rho_{33} 
\end{bmatrix}
\begin{bmatrix}
  J_1 \\
  J_2 \\
  J_3 
\end{bmatrix}
\] (1)

or using the summation convention:

\[
E_i = \rho_{ij} J_j, \quad i,j = 1, 2, 3
\] (2)

where the indices \( i, j \) represent the orthogonal basis vector directions \( e_1, e_2 \) and \( e_3 \). The piezoresistive behavior is described using the Bridgman piezoresistivity model [32]:

\[
\rho_{ij} = \rho_{0ij} + d\rho_{ij} = \rho_{0ij} \left( 1 + \frac{d\rho_{ij}}{\rho_{0ij}} \right), \quad i,j = 1, 2, 3.
\] (3)

where \( \rho_{0ij} \) represents the initial resistivity when no mechanical load is applied to the specimen and \( d\rho_{ij} \) is the increment of the resistivity due to the mechanical load. In general, the initial resistivity \( \rho_{0ij} \) is temperature dependent. However, in this research a constant temperature is assumed. The relative resistivity change \( d\rho_{ij}/\rho_{0ij} \) linearly depends on the strain \( \varepsilon_{kl} \) as [33]:

\[
d\rho_{ij}/\rho_{0ij} = \xi_{ijkl} \varepsilon_{kl}, \quad i,j,k,l = 1, 2, 3
\] (4)

where \( \xi_{ijkl} \) represents the piezoresistive coefficient. Using Voigt-Kelvin notation, which simplifies the two-subscript notation into a single-subscript
notation [34]:

\[ 11 \rightarrow 1, \quad 22 \rightarrow 2, \quad 33 \rightarrow 3, \quad 23 \rightarrow 4, \quad 13 \rightarrow 5, \quad 12 \rightarrow 6 \]  

(5)

Eq. (4) simplifies to

\[ \frac{d\rho_i}{\rho_0 i} = \xi_{ij} \varepsilon_j, \quad i, j = 1, ..., 6. \]  

(6)

Under the assumption of a linear elasticity hypothesis, the strain and stress \( \sigma_j \) relationship is equal to [34]:

\[ \varepsilon_i = s_{ij} \sigma_j, \quad i, j = 1, ..., 6 \]  

(7)

where \( s_{ij} \) denotes the compliance coefficients. As a result, the relative resistivity change can be written in terms of the stress:

\[ \frac{d\rho_i}{\rho_0 i} = \xi_{ij} s_{jk} \sigma_k = \pi_{ik} \sigma_k, \quad i, j, k = 1, ..., 6 \]  

(8)

where \( \pi_{ik} \) represents the piezoresistive coefficient relating the relative resistivity change \( d\rho_i/\rho_0 i \) and the stress \( \sigma_k \). When the structures are manufactured with material being deposited in a single direction, as shown in Fig. 1 a), the symmetry of the material properties in three orthogonal planes is assumed (along the deposited material, and transverse in the horizontal and vertical planes) [35, 36], resulting in orthotropic material properties:

\[ \pi = \begin{bmatrix}
\pi_{11} & \pi_{12} & \pi_{13} & 0 & 0 & 0 \\
\pi_{21} & \pi_{22} & \pi_{23} & 0 & 0 & 0 \\
\pi_{31} & \pi_{32} & \pi_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & \pi_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & \pi_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & \pi_{66}
\end{bmatrix}, \quad \rho = \begin{bmatrix}
\rho_1 & 0 & 0 \\
0 & \rho_2 & 0 \\
0 & 0 & \rho_3
\end{bmatrix}. \]  

(9)

The article is focused on a special case, when the electric field intensity \( E \), the electric current density \( J \) and the stress \( \sigma \) are in the directions of the material symmetry as shown in Figs. 1 b), c) and d). Ohm’s law and the relative resistivity change for the case shown in Fig. 1 b) are:

\[ \frac{E_1}{\rho_1} = \rho_1 \frac{J_1}{}, \]  

(10)

\[ \frac{d\rho_1}{\rho_0} = \pi_{11} \sigma_1. \]  

(11)
Ohm’s law and the relative resistivity change for the case shown in Fig. 1 c):

\[
E_2 = \rho_2 J_2,
\]
\[
\frac{d\rho_2}{\rho_0} = \pi_{22} \sigma_2
\]

and for Fig. 1 d):

\[
E_3 = \rho_3 J_3,
\]
\[
\frac{d\rho_3}{\rho_0} = \pi_{33} \sigma_3.
\]

Equations (10) - (15) show that the piezoresistivity in material’s principle directions \(\pi_{11}, \pi_{22}\), and \(\pi_{33}\) can be identified by establishing the stress and electric fields in the material’s principle directions.

Figure 1: a) Unidirectional FFF structure; Homogenised equivalent with b) electric field instensity \(E\), current density \(J\) and stress \(\sigma\) in the direction \(e_1\), c) \(E, J\), and \(\sigma\) in the direction \(e_2\) and d) \(E, J\), and \(\sigma\) in the direction \(e_3\)
3. Method

3.1. Specimens
The specimen orientation with regard to the load cases shown in Figs. 1 b), c) d) are shown in Fig. 2 as O1, O2 and O3, respectively. The dimensions of the specimens are \( l_1 = 110 \text{ mm} \), \( l_2 = 20 \text{ mm} \) and \( l_3 = 7 \text{ mm} \). A harmonic mechanical excitation with no offset is applied to the specimens. Due to the compressive phase of the excitation, buckling should be prevented. The critical buckling load that should not be exceeded is approximately 290 N (for a Young’s modulus of 2 GPa).

![Figure 2: Specimen: arrows indicate mechanical load direction](image)

3.2. Electrical contacts
The electrical contact preparation is shown in Fig. 3. In the first step, the surface of the electrically conductive specimen is coated with a silver paint [37] and left to dry. In the second step, a thin enamelled copper wire is soldered to the conductive copper tape [38], which is then taped to the specimen at the location of the silver paint. A relatively thin wire has to be used, to prevent any significant contribution to the specimen’s dynamics.
3.3. Piezoresistivity identification

Fig. 4 shows the proposed experimental setup for the identification of the piezoresistivity. The specimen is mechanically fixed on one side, while it is able to move in the axial direction on the other side. In this section, if not otherwise noted, the orientation of specimen O1 is assumed, see Fig. 2. Dimension $l_C = 30$ mm denotes the clamping length.

For the identification of the electrical resistivity, the four-probe method [39] is used to measure the specimen’s resistance. The electrical contacts have
the electric potentials $V_1$, $V_2$, $V_3$ and $V_4$. The contacts $V_3$ and $V_4$ are $l_E = 10$ mm apart and are assumed to be in the electrical and mechanical homogeneous zone without boundary effects (e.g., due to clamping, electrodes, for details see Appendix A). The supply circuit is connected to the contacts $V_1$ and $V_2$, which are $l_1$ apart. The electrical current through the specimen $i$ is determined through the voltage drop across the shunt resistor $u_S$ with resistance $R_S$, see Fig. 4:

$$i = \frac{u_S}{R_S}$$ \hfill (16)

By measuring the voltage drop between the contacts $V_3$ and $V_4$, the electrical resistance of the 3D printed structure between the contacts is determined as:

$$R = \frac{V_3 - V_4}{i} = \frac{u_R}{i}. \hfill (17)$$

Due to the geometrical relationship between the resistance $R$ and the resistivity $\rho_1$ it follows that [40]:

$$\rho_1 = R \frac{(l_2 + dl_2)(l_3 + dl_3)}{l_E + dl_E} = \frac{u_R}{i} \frac{(l_2 + dl_2)(l_3 + dl_3)}{l_E + dl_E}, \hfill (18)$$

where $dl_2$, $dl_3$, $dl_E$ denote the length changes of $l_2$, $l_3$ and $l_E$ due to the applied load, respectively. Under a uniaxial stress assumption, the strain equals:

$$\varepsilon_1 = \frac{dl_E}{l_E}, \quad \varepsilon_2 = \frac{dl_2}{l_2} = -\nu_{12} \varepsilon_1, \quad \varepsilon_3 = \frac{dl_3}{l_3} = -\nu_{13} \varepsilon_1, \hfill (19)$$

and equation (18) simplifies to:

$$\rho_1 = R \frac{l_2 l_3}{l_E} \left( \frac{1 - \nu_{12} \varepsilon_1 (1 - \nu_{13} \varepsilon_1)}{1 + \varepsilon_1} \right), \hfill (20)$$

where $\nu_{12}$ and $\nu_{13}$ denote the Poisson’s ratio relating the axial strain $\varepsilon_1$ with the transverse strain $\varepsilon_2$ and $\varepsilon_3$, respectively. The influence of the geometrical changes in Eq. (20) are discussed in Appendix B.

This research is focused on the harmonic changes in the resistivity when
a harmonic load $F(t)$ is applied to the moving end, see Fig. 4:

$$F(t) = \tilde{F} e^{i2\pi f_{\text{exc}} t},$$

(21)

where $\tilde{F}$ denotes the force amplitude, $i$ is an imaginary number, $\pi$ is the number Pi, $f_{\text{exc}}$ is the excitation frequency in Hz and $t$ is time. For a causal, linear system, the harmonic force causes a harmonic displacement $x_1(t)$, a stress $\sigma_1(t)$, strain $\varepsilon_1(t)$ and also a resistivity $\rho_1(t)$ in the axial direction. The strain in the axial direction $\varepsilon_1(t)$ is estimated from the displacement $x_1(t)$:

$$\varepsilon_1(t) = \frac{x_1(t)}{l_1 - 2l_C},$$

(22)

and the Poisson’s ratios $\nu_{12}$ and $\nu_{13}$ (20) can be estimated from a static tensile test or by simultaneous measurements of the transverse strain $\varepsilon_2(t)$ and $\varepsilon_3(t)$. The measured resistivity $\rho_1$ (3)(20):

$$\rho_1(t) = \rho_{01} + d\rho_1(t).$$

(23)

is related to the underlying resistivity coefficient via Eq. (11); however, due to the assumed constant temperature, $\rho_{01}$ is related to the mean value $\rho_{01} = \text{Mean}(\rho_1)$ and the changes in resistivity $d\tilde{\rho}_1$ to the amplitude at the excitation frequency $f_{\text{exc}}$. The amplitude spectrum $\tilde{\rho}_1(f)$ is obtained by transforming the resistivity $\rho_1(t)$ into the frequency domain using a Fourier transform. The resistivity $\rho_{01}$ and the resistivity change $d\tilde{\rho}_1$ are obtained as:

$$\rho_{01} = \tilde{\rho}_1(f = 0),$$

(24)

$$d\tilde{\rho}_1 = \tilde{\rho}_1(f = f_{\text{exc}}).$$

(25)

At the excitation frequency $f_{\text{exc}}$, due to the harmonic response, the piezoresistivity coefficient $\pi_{11}$ (11) can be related to the amplitude of the change in resistivity $\tilde{\rho}_1(f_{\text{exc}})$, the amplitude in the mechanical stress $\tilde{\sigma}(f_{\text{exc}})$ and the initial resistivity $\tilde{\rho}_1(f = 0)$:

$$\pi_{11}(f_{\text{exc}}) = \frac{\tilde{\rho}_1(f_{\text{exc}})}{\tilde{\rho}_1(f = 0) \tilde{\sigma}(f_{\text{exc}})}.$$
The amplitude of the mechanical stress $\tilde{\sigma}(f_{\text{exc}})$ is defined as, see Fig 2:

$$\tilde{\sigma}(f_{\text{exc}}) = \frac{\tilde{F}_R(f_{\text{exc}})}{l_2 l_3}, \quad (27)$$

where $\tilde{F}_R(f_{\text{exc}})$ is the amplitude of the force in the specimen, which can be related to the measured force amplitude $\tilde{F}(f_{\text{exc}})$:

$$\tilde{F}_R(f_{\text{exc}}) = \tilde{F}(f_{\text{exc}}) - m \ddot{x}(f_{\text{exc}}), \quad (28)$$

where $m$ is the moving mass and $\ddot{x}$ is the measured acceleration amplitude of the moving mass. Further, the moving mass $m$ is estimated as:

$$m = m_C + m_S \left(1 - \frac{l_C}{l_1}\right), \quad (29)$$

where $m_C = 0.403\, \text{kg}$ is the mass of the fixation clamps at one side and $m_S \approx 0.01\, \text{kg}$ is the mass of the specimen. As a reasonable approximation, in Eq. (29) the complete mass of the specimen except the clamped length $l_C$ was assumed to be moving.

To summarize: based on the measured values $F(t)$, $\ddot{x}(t)$, $u_S(t)$, $u_R(t)$, the coefficient of piezoresistivity for the specimen orientation O1, i.e., $\pi_{11}$ Eq (26), can be identified. The specimen orientations O2 and O3 can be used for the identification of the piezoresistivity coefficients $\pi_{22}$ and $\pi_{33}$, respectively. For the O2 and O3 orientations, the resistivity in Eq. (26) changes as follows:

$$O2, \pi_{22} : \quad \tilde{\rho}_1 \rightarrow \tilde{\rho}_2 \quad (30)$$
$$O3, \pi_{33} : \quad \tilde{\rho}_1 \rightarrow \tilde{\rho}_3 \quad (31)$$

4. Experiment

4.1. Specimen preparation

Specimens with the orientations O1, O2 and O3 (Fig. 2) were 3D printed with a PRUSA I3 MK3S. Two specimens for each orientation were manufactured, resulting in a total of 6 specimens. An electrically conductive composite filament from Proto-pasta [41] with a diameter of 1.75 mm was used. The conductive Proto-pasta filament consists of a non-conductive PLA matrix and conductive carbon-black particles. Based on the studies of Maurizi et al. [15] and Munasinge [42], a linear piezoresistive behaviour in $\varepsilon < 0.5\%$ is assumed. The printing parameters were layer height 0.15 mm,
infill density 100%, printing temperature of the conductive PLA 225°C, built plate temperature 70°C, line width 0.4 mm, lines as infill pattern.

4.2. Experimental setup

The experimental setup is shown in Fig. 5. The specimen is clamped between one fixed clamp attached to the support structure and one clamp attached to a LDS V555 eletrodynamical shaker. In order to electrically insulate the specimen from the environment, a thin insulating tape at the interface between the specimen and clamping was used. The acceleration and force from Fig. 4 were measured with a PCB T333B30 accelerometer and a PCB 218C charge force sensor, which were mounted between the shaker and the moving clamp. With a force sensor the Brüel and Kjær Nexus 2692 charge pre-amplifier was used. The signal from the accelerometer and force sensor were acquired using a National Instruments 9234 measuring card.

A constant voltage was supplied to the specimen using a HQ-Power PS23023 adjustable DC power supply. The voltage drop across the current shunt and specimen, see Fig.4, were acquired using a National Instruments 9215 measuring card.
4.3. Measurements

The specimen was supplied with a constant voltage and current of approximately 10 mA. The electrodynamic shaker excited the specimen with a displacement amplitude of 40 µm at selected frequencies (55 Hz, 110 Hz, 165 Hz and 220 Hz) for 30 s. The excitation frequencies were significantly below the first natural frequency of the system, which was identified in the range 500-550 Hz (depending on the specimen orientation). The displacement amplitude of 40 µm causes a normal strain of 0.08 %, which is assumed to be in the linear region. When linearity cannot be assumed, the piezoresistive coefficients at different displacement amplitudes have to be determined to find the linear region. The identification of the dynamic piezoresistivity (26) is based on the simultaneously measured force $F(t)$, acceleration $\ddot{x}(t)$, voltage drops $u_R(t)$ and $u_S(t)$. The voltage drops $u_R(t)$ and $u_S(t)$ were used to determine the resistivity $\rho(t)$ (20). Poisson’s ratios $\nu_{12} = 0.37$ and $\nu_{13} = 0.38$ were determined for the O1 oriented specimen using a static tensile test. Poisson’s ratio $\nu_{12} = \nu_{13} = \nu = 0.375$ was used
for the geometrical correction in Eq. (20).

The measured force \( F(t) \), acceleration \( \ddot{x}(t) \) and resistivity \( \rho(t) \) were, using a Fourier transform, transformed from the time to the frequency domain. A particular 30-s-long time-domain signal was, due to averaging, divided into 30 non-overlapping segments of length 1 s (a Hanning window was used). Python library SciPy [43] was used for signal processing. The resulting amplitude complex spectra are \( \tilde{F}(f) \), \( \tilde{\dot{x}}(f) \), \( \tilde{\rho}(f) \), see, e.g., Fig. 6 for the O3-oriented specimen at \( f_{\text{exc}} = 220 \) Hz excitation frequency. From the amplitude spectra, complex amplitudes at the excitation frequency \( \tilde{F}(f_{\text{exc}}) \), \( \tilde{\dot{x}}(f_{\text{exc}}) \), \( \tilde{\rho}(f_{\text{exc}}) \) and static resistivity \( \tilde{\rho}(f = 0) \) were obtained to identify the piezoresistive coefficient, see Fig. 6 and Eqs. (26), (27), (28).

5. Results

Time- and frequency-domain results for the specimen orientation O3 at 220 Hz harmonic excitation are shown in Fig. 6. The major findings from the results are:

- Static resistivity \( \tilde{\rho}(f = 0) \approx 34.3 \) Ohm cm is approximately three orders of magnitude higher than the harmonic resistivity at the excitation frequency \( \tilde{\rho}(f_{\text{exc}}) \) which is further approximately three orders of magnitude above the noise floor.

- Stress in the specimen \( \tilde{\sigma}(f_{\text{exc}}) \) is based on the amplitude of the excitation force \( \tilde{F}(f_{\text{exc}}) \) and acceleration \( \tilde{\dot{x}}(f_{\text{exc}}) \). Force and acceleration amplitudes are approximately five and four orders of magnitude above the noise floor, respectively. Both are contaminated with electromagnetic noise (especially at 100 Hz).

Similar results were obtained for other specimen orientations and excitation frequencies.
In Tab. 1 the identified mean amplitudes at measured excitation frequencies (55 Hz, 110 Hz, 165 Hz, 220 Hz) and the piezoresistive coefficient for O1, O2, O3 specimen orientations are presented. The specimens exhibited a low viscoelastic behaviour (phase shift of less than 3 °). Due to the low impact on the final result (less than 1 %), absolute values with the appropriate sign (+/-) are used for calculations of the results in Tab 1, instead of complex amplitudes. Based on the results in Tab 1, the following observation were made:

- Initial resistivity \( \rho_0 = \tilde{\rho}(0) \) does not depend on the frequency with \( \rho_{01} = 20.96 \pm 1.21 \) Ohm cm, \( \rho_{02} = 32.96 \pm 0.62 \) Ohm cm, \( \rho_{03} = 33.83 \pm 1.08 \) Ohm cm.

- Amplitude of acceleration is constant at a certain frequency point and negative, as the system is operating below the resonance frequency.

- Force \( \tilde{F}(f_{\text{exc}}) \) decrease with frequency.
The highest piezoresistive coefficient is exhibited by orientation O3 \( \pi_{33} = 3.54 \pm 0.11 \text{ GPa}^{-1} \), followed by O2 \( \pi_{22} = 2.30 \pm 0.41 \text{ GPa}^{-1} \) and O1 \( \pi_{11} = 0.17 \pm 0.05 \text{ GPa}^{-1} \).

Table 1: Identified average amplitudes used for calculation of the piezoresistive coefficient and identified piezoresistive coefficient for different specimen orientations and excitation frequencies

<table>
<thead>
<tr>
<th>orientation</th>
<th>( f ) [Hz]</th>
<th>specimen</th>
<th>( \bar{\rho}(0) ) [Ohm cm]</th>
<th>( \bar{\rho}(f_{exc}) ) [Ohm mm]</th>
<th>( F(f_{exc}) ) [N]</th>
<th>( x(f_{exc}) ) [m s(^2)]</th>
<th>( \pi(f_{exc}) ) [GPa(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1</td>
<td>55</td>
<td>1</td>
<td>21.54</td>
<td>0.04</td>
<td>172.06</td>
<td>-4.81</td>
<td>0.15</td>
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<tr>
<td></td>
<td></td>
<td>2</td>
<td>20.20</td>
<td>0.05</td>
<td>173.95</td>
<td>-4.81</td>
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<td></td>
<td>110</td>
<td>1</td>
<td>21.61</td>
<td>0.03</td>
<td>163.00</td>
<td>-19.18</td>
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6. Discussion

Due to the lack of research, the piezoresistive coefficients \( \pi \) are hard to compare to previous experiments; however, the initial resistivity \( \rho_0 \) can be compared to previous research, see Tab. 2. The results are in reasonable agreement with the research of Watschke et al. [24] and Hampel et al. [22]. The resistivity in the deposition direction \( \rho_{01} \) in comparison with \( \rho_{02} \) and \( \rho_{03} \) is smaller due to the additional resistivity of the adjacent traces for \( \rho_{02} \) and \( \rho_{03} \). Hampel et al. examined the increase of the resistivity due to the
adjacent traces and found that it is the same for horizontal and vertical (build-up) directions. This finding by Hampel et al. is similar to that reported here \( \rho_{02} \approx \rho_{03} \).

Table 2: Initial resistivity comparison

<table>
<thead>
<tr>
<th>Initial resistivity [\Omega \text{cm}]</th>
<th>This research, Tab. 1</th>
<th>Watschke et al. [24]</th>
<th>Hampel et al. [22]</th>
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<tr>
<td>( \rho_{01} )</td>
<td>20.96</td>
<td>6 (&lt;\rho_{1} )&lt;16</td>
<td>4.94</td>
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<tr>
<td>( \rho_{02} )</td>
<td>32.96</td>
<td>11 (&lt;\rho_{2} )&lt;24</td>
<td>/</td>
</tr>
<tr>
<td>( \rho_{03} )</td>
<td>33.83</td>
<td>/</td>
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While there is no significant difference of the initial resistivity in the directions \( \rho_{02} \) and \( \rho_{03} \), the piezoresistive coefficients \( \pi_{22} \) and \( \pi_{33} \) differ significantly. This can be attributed to the different mechanical bonding strength of the adjacent traces. Since the FFF parts are built layer by layer a lower temperature field appears when the material is deposited in the vertical direction due to the cooling of already-deposited material. A lower temperature results in a lower bonding strength in the vertical direction, as compared to the horizontal plane, see [44], [45]. Weaker mechanical bonding could cause the conductive networks to break and form more easily, resulting in a higher piezoresistive coefficient \( \pi_{33} \).

7. Conclusions

A method for the identification of the dynamic piezoresistive coefficients of unidirectional FFF structures was proposed. The method is based on the Bridgman model of piezoresistivity and is extended to harmonic mechanical excitation. Three build orientations of the specimens were used to identify the piezoresitive properties in three orthogonal directions. Based on the simultaneous measurement of force, acceleration and voltage drops across the specimen and the shunt resistor the dynamic piezoresistive coefficient can be identified.

The method was successfully implemented on three orthogonally manufactured specimens in the 55 Hz-220 Hz frequency range. Significantly different piezoresistive coefficients were identified: \( \pi_{11} = 0.17 \pm 0.05 \text{GPa}^{-1} \), \( \pi_{22} = 2.30 \pm 0.41 \text{GPa}^{-1} \) and \( \pi_{33} = 3.54 \pm 0.11 \text{GPa}^{-1} \) for the O1, O2 and O3 orientations, respectively. In the researched frequency range, the piezoresistive coefficients were found to be constant.

The identified piezoresistive coefficients in different directions enable the future numerical and analytical research on advance dynamic sensors with an arbitrary spatial orientation of embedding.
Acknowledgements

The authors acknowledge the partial financial support from the Slovenian Research Agency (research core funding No. P2-0263).

Appendix A. Effects of the finite length of the electrodes on the identified resistivity

Electrical contacts with voltages $V_1$ and $V_2$ (see Fig. 5) are applied to the $l_2-l_3$ faces. As a result, a homogenous electric field can be assumed in the axial direction. In order to avoid the effects of clamping, the voltage drops between the electrodes $V_3$ and $V_4$ are measured to determine the resistivity in Sec. 3.3. However, the finite length of the electrodes $V_3$, $V_4$, can influence the measured resistivity. To check whether the electrodes $V_3$ and $V_4$ have a significant influence on the measurement, three resistivity measurements on the O1 oriented specimen were performed. Figure A.7 shows the preparation of the specimen. In the first step, only the contacts $V_1$ and $V_2$ were prepared, as described in 3.2 and shown in Fig. A.7 a). The resistivity $\rho_{M1}$ was determined from the voltage drop between $V_1$ and $V_2$

$$\rho_{M1} = \frac{V_1 - V_2}{i} \frac{l_2 l_3}{l_1}. \quad (A.1)$$

In the second step, the electrical contacts $V_3$ and $V_4$ were prepared, see Fig. A.7 b). The resistivity $\rho_{M2}$ was determined from the voltage drop between $V_1$ and $V_2$ using Eq. (A.1). Lastly, the resistivity $\rho_{M3}$ was determined from voltage drop between $V_3$ and $V_4$ as:

$$\rho_{M3} = \frac{V_3 - V_4}{i} \frac{l_2 l_3}{l_E}. \quad (A.2)$$

The measured resistivities were $\rho_{M1} = 21.9$ Ohm cm, $\rho_{M2} = 21.8$ Ohm cm and $\rho_{M3} = 21.4$ Ohm cm. Based on the results, the impact of the electrodes is in the range of approximately 2% and was neglected in this study.
Appendix B. The effects of the correction of geometrical changes on the identified piezoresistivity

In Eq. (20) the resistivity is determined by taking into account the geometrical changes:

\[ g = \frac{(1 - \nu_{12} \epsilon_1)(1 - \nu_{13} \epsilon_1)}{1 + \epsilon_1}. \]  \hspace{1cm} (B.1)

The effect of neglecting the geometrical changes \( g \) is presented here on a synthetic experiment. Assuming a harmonic stress \( \sigma(t) \) acting on a specimen in the axial direction:

\[ \sigma = \bar{\sigma} \cos(2\pi f_{exc} t), \] \hspace{1cm} (B.2)

causes, under the assumption of negligible viscoelastic behaviour, a harmonic resistivity change \( d\rho(t) \) and a strain in the axial direction \( \varepsilon(t) \):

\[ d\rho(t) = \pi \bar{\sigma} \cos(2\pi f_{exc} t) = d\bar{\rho}(t) \cos(2\pi f_{exc} t), \] \hspace{1cm} (B.3)

\[ \varepsilon(t) = s \bar{\sigma} \cos(2\pi f_{exc} t) = \ddot{\varepsilon} \cos(2\pi f_{exc} t). \] \hspace{1cm} (B.4)
By introducing the ratio $r$ between the resistivity change amplitude $d\tilde{\rho}$ and the strain amplitude $\tilde{\varepsilon}$

$$r = \frac{d\tilde{\rho}}{\tilde{\varepsilon}},$$

(B.5)

and rearranging Eqs. (20), (23), (B.3), (B.4), resistivity not corrected for geometrical changes $h(t)$ is:

$$h(t) = R(t) \frac{l_2 l_3}{l_E} = \left[\rho_0 + r \tilde{\varepsilon} \cos(2 \pi f_{\text{exc}} t)\right] \frac{1 + \tilde{\varepsilon} \cos(2 \pi f_{\text{exc}} t)}{(1 - \nu \tilde{\varepsilon} \cos(2 \pi f_{\text{exc}} t))^2}$$

(B.6)

where the Poisson’s ratios in different directions are assumed to be equal $\nu = \nu_{12} = \nu_{13}$. When the geometrical changes are neglected, $h(t)$ represents the time signal used for the identification of the initial resistivity $\rho_{0I}$ and the amplitude of resistivity change $d\tilde{\rho}_I$. Index I denotes the identified quantities. Transforming $h(t)$ into the frequency-domain $\tilde{h}(f)$ using a Fourier transform, the initial resistivity and the resistivity change are determined from Eqs. (24), (25) as:

$$\rho_{0I} = \tilde{h}(f = 0),$$  

(B.7)

$$d\tilde{\rho}_{0I} = \tilde{h}(f_{\text{exc}}).$$  

(B.8)

The synthetic function $h(t)$ was generated from the parameters $f_{\text{exc}} = 55$ Hz, $\rho_0 = 0.1 \Omega$m, $\tilde{\varepsilon} = 10^{-3}$, $\nu = 0.25$ and focusing the analysis on the interval $r \in (10^{-6}, 10^4) \Omega$ m. In this synthetic experiment, for different ratios $r$, the amplitude of the resistivity change $d\tilde{\rho}_I$ was identified. In Fig. B.8 a) the identified increment $d\tilde{\rho}_I$ normalised by the true increment $d\tilde{\rho}$ is shown. For low ratios, the geometrical changes predominantly cause a change of the resistance. As a result, significantly higher resistivity increments are identified. When the ratio $r$ increases, the resistivity change is the main cause of the resistance change. Additionally, the parameters $\varepsilon$, $\nu$ and $\rho_0$ were varied. A negligible influence of the strain variations was observed. A slight influence of the Poisson’s ratio $\nu$ variations and a significant influence of the initial resistivity $\rho_0$ variation were observed, see Fig. B.8 c) and b), respectively. The synthetic experiment shows that the geometrical changes should not be neglected, since a significant influence on the identified resistivity was observed.
Figure B.8: The influence of ratio \( r = \frac{d\tilde{\rho}}{\tilde{\varepsilon}} \) on the ratio between identified increment of resistivity \( d\tilde{\rho}_I \) and true increment of resistivity \( d\tilde{\rho} \) when parameters a) \( f_{\text{exc}} = 55 \, \text{Hz}, \rho_0 = 0.1 \, \Omega\text{m}, \tilde{\varepsilon} = 10^{-3}, \nu = 0.25 \), b) \( f_{\text{exc}} = 55 \, \text{Hz}, \tilde{\varepsilon} = 10^{-3}, \nu = 0.25 \), c) \( f_{\text{exc}} = 55 \, \text{Hz}, \rho_0 = 0.1 \, \Omega\text{m}, \tilde{\varepsilon} = 10^{-3} \) are used.
References


