Univerza v Ljubljani
Fakulteta za strojništvo

Katedra za strojne elemente
in razvojna vrednotenja

N·VE·K
LABORATORIJ ZA VREDNOTENJE KONSTRUKCIJ

L·N·S·E·M
LABORATORIJ ZA STROJNE ELEMENTE
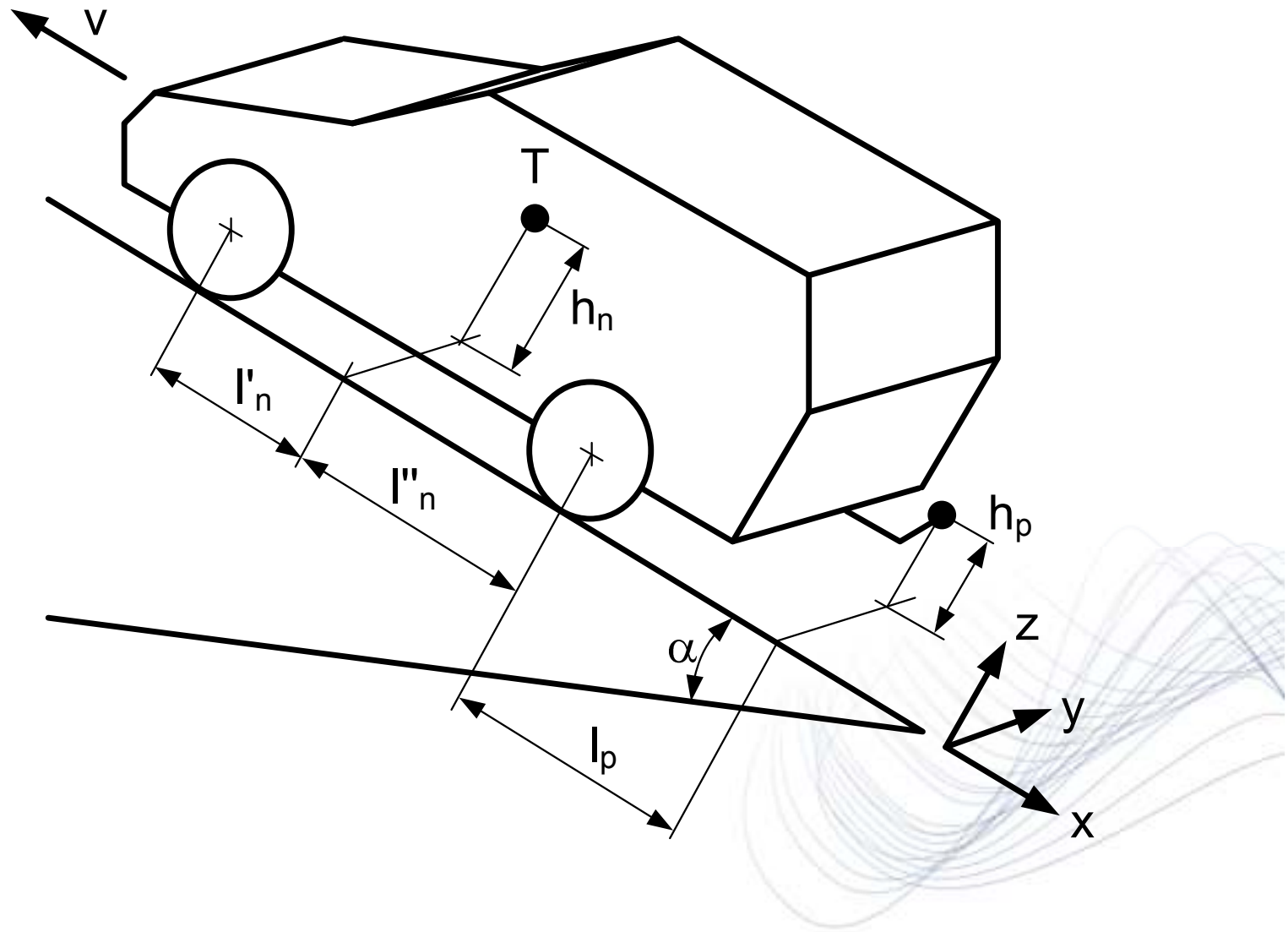
Gibalna enačba vozila

Vehicle's equation of motion
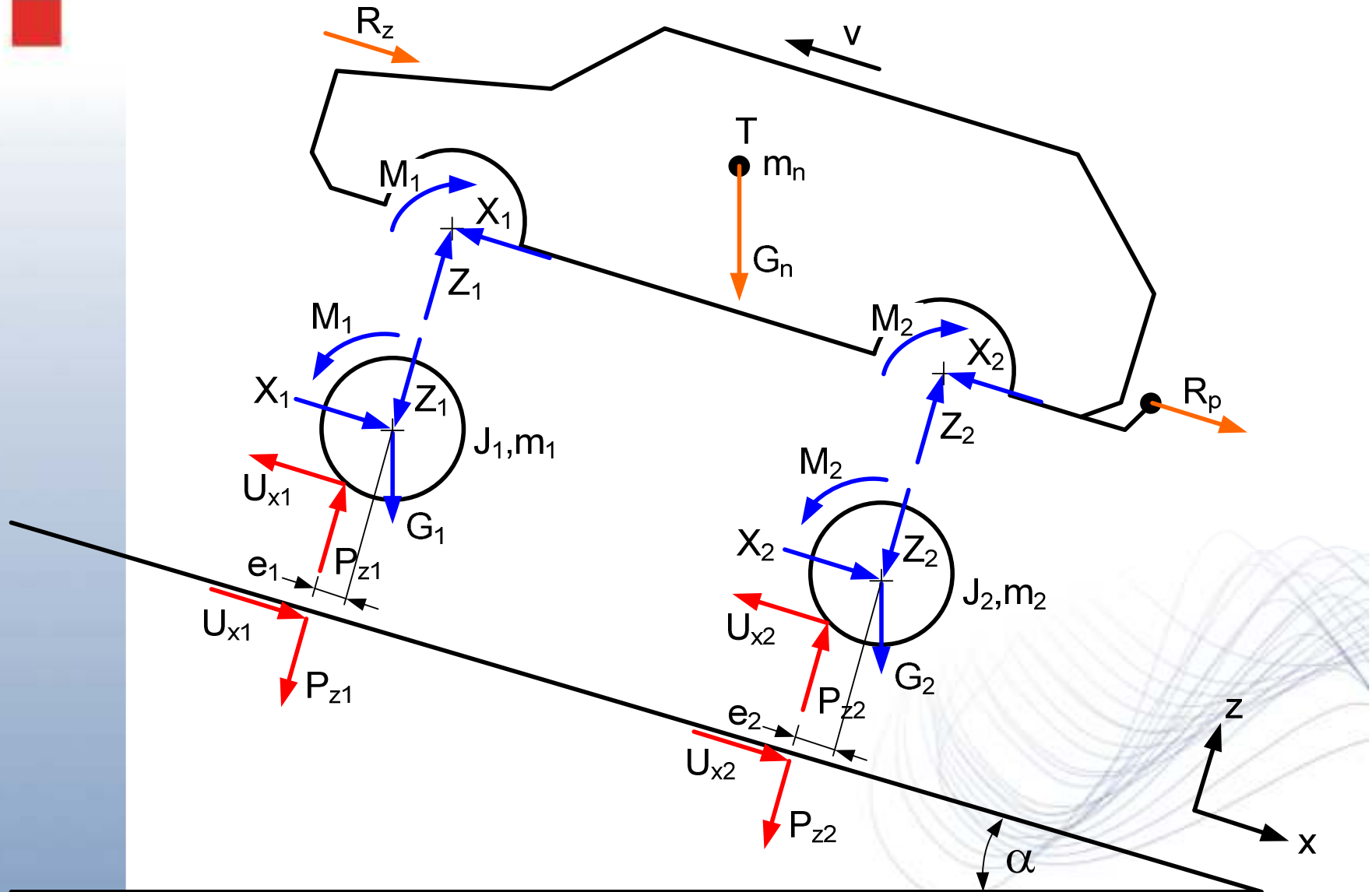
Prof. dr. Jernej Klemenc

Koordinatni sistem vozila /
Coordinate system of a vehicle

Sile na vozilo v X-Z ravnini /
Forces that act on a vehicle in a X-Z plane

Sistem ravnotežnih enačb za vozilo / *System of vehicle's equations of balance*

Gibalne enačbe za nadgradnjo /
Chassis equatios of motion:

(1) $\quad \sum F_{x,i} = m_n \cdot \ddot{x}_n = -G_n \cdot \sin\alpha + X_1 + X_2 - R_z - (R_p)$

(2) $\quad \sum F_{z,i} = 0 = Z_1 + Z_2 - G_n \cdot \cos\alpha$

(3) $\quad \sum M_i = 0 = -(M_1 + M_2) - Z_1 \cdot l'_n + Z_2 \cdot l''_n -$
$\quad - X_1 \cdot (h_n - r_{st1}) - X_2 \cdot (h_n - r_{st2}) + \left[R_p \cdot (h_n - h_p) \right]$

Gibalne enačbe za prvo podvozje /
Font-axle equatios of motion:

(4) $\quad \sum F_{x,i} = m_1 \cdot \ddot{x}_1 = U_{x1} - X_1 - G_1 \cdot \sin\alpha$

(5) $\quad \sum F_{z,i} = 0 = P_{z1} - Z_1 - G_1 \cdot \cos\alpha$

(6) $\quad \sum M_i = J_1 \cdot \ddot{\varphi}_1 = M_1 - U_{x1} \cdot r_{st1} - P_{z1} \cdot e_1$
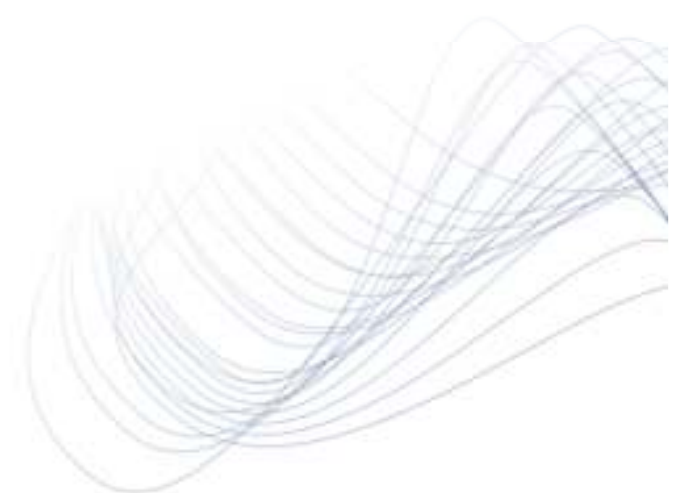
Sistem ravnotežnih enačb za vozilo / *System of vehicle's equations of balance*

<u>Gibalne enačbe za drugo podvozje /</u>
<u>*Rear-axle equatios of motion:*</u>

(7) $\quad \sum F_{x,i} = m_2 \cdot \ddot{x}_2 = U_{x2} - X_2 - G_2 \cdot \sin \alpha$

(8) $\quad \sum F_{z,i} = 0 = P_{z2} - Z_2 - G_2 \cdot \cos \alpha$

(9) $\quad \sum M_i = J_2 \cdot \ddot{\varphi}_2 = M_2 - U_{x2} \cdot r_{st2} - P_{z2} \cdot e_2$

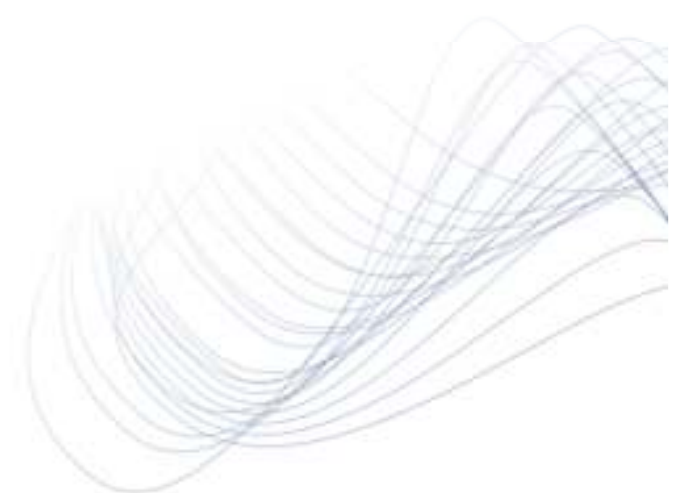# Sistem ravnotežnih enačb za vozilo / *System of vehicle's equations of balance*

(2) + (5) + (8):
$$P_{z1} + P_{z2} = (G_n + G_1 + G_2) \cdot \cos\alpha$$

$$G_n + G_1 + G_2 = G$$

(6):
$$\frac{J_1 \cdot \ddot{\varphi}_1}{r_{st1}} = \frac{M_1}{r_{st1}} - U_{x1} - \underbrace{P_{z1} \cdot \frac{e_1}{r_{st1}}}_{R_{f1}}$$

$$\frac{J_1 \cdot \ddot{\varphi}_1}{r_{st1}} = \frac{M_1}{r_{st1}} - U_{x1} - R_{f1}$$

(9):
$$\frac{J_2 \cdot \ddot{\varphi}_2}{r_{st2}} = \frac{M_2}{r_{st2}} - U_{x2} - \underbrace{P_{z2} \cdot \frac{e_2}{r_{st2}}}_{R_{f2}}$$

$$\frac{J_2 \cdot \ddot{\varphi}_2}{r_{st2}} = \frac{M_2}{r_{st2}} - U_{x2} - R_{f2}$$

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Gibalna enačba vozila /
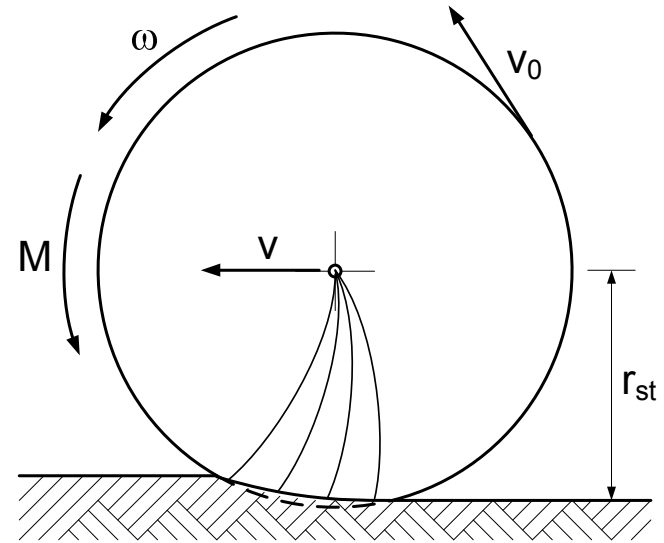# *Vehicle's equation of motion*

(1) + (4) + (6) + (7) + (9):

$$r_{st} \cdot \omega = v_0 < v$$

$$\sigma_t = \frac{v - v_0}{v_0} \Rightarrow v = \omega \cdot r_d$$

$$\ddot{\varphi}_i = \frac{\ddot{x}_i}{r_{d,i}}; \; x_n = x_1 = x_2$$
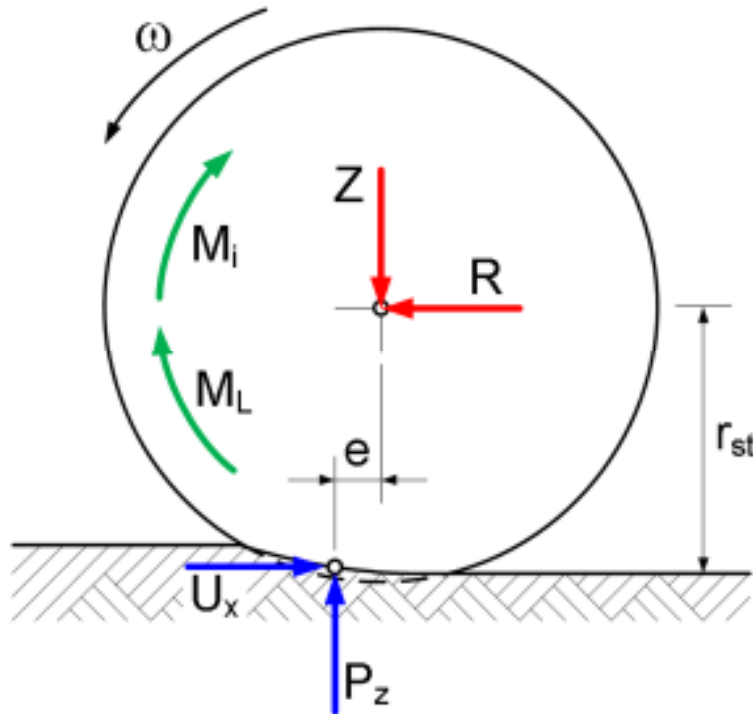
$$m_n + m_1 + m_2 = m = G / g$$

$$\frac{M_1}{r_{st1}} + \frac{M_2}{r_{st2}} = F_{K1} + F_{K2} =$$

$$= \left[\frac{G}{g} + \frac{J_1}{r_{st1} \cdot r_{d1}} + \frac{J_2}{r_{st2} \cdot r_{d2}} \right] \cdot \ddot{x} + G \cdot \sin \alpha + R_z + \underbrace{R_{f1} + R_{f2}}_{R_f} + (R_p)$$

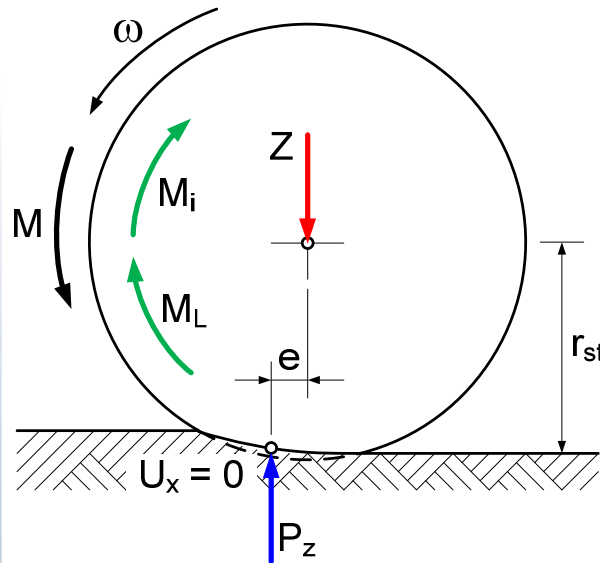# Sile in navori na potiskanem kolesu / *Forces and torques on a non-traction wheel*

M_i ... Navor za premagovanje vztrajnostni kolesa / *Inertia equivalent torque*

M_L ... Navor za premagovanje uporov v ležajih, venti-lacijskih izgub itn. / *Torque losses in bearings, due to ventilation etc.*

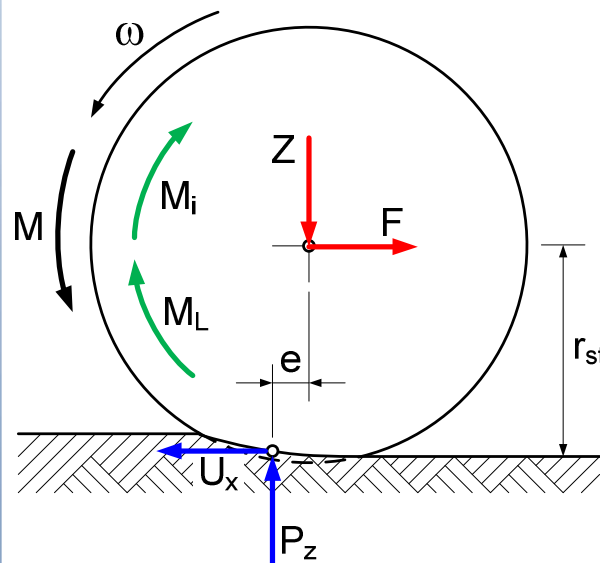$$R = U_x = Z \cdot \underbrace{\frac{e}{r_{st}}}_{R_f} + \frac{M_L}{r_{st}} + \frac{M_i}{r_{st}}$$

Sile in navori na gnanem kolesu /
Forces and torques on a traction wheel

Pogonski navor kompenzira upore /
Driving torque compensates losses:

$$\frac{M}{r_{st}} = \underbrace{Z \cdot \frac{e}{r_{st}}}_{R_f} + \frac{M_L}{r_{st}} + \frac{M_i}{r_{st}}; U_x = 0$$
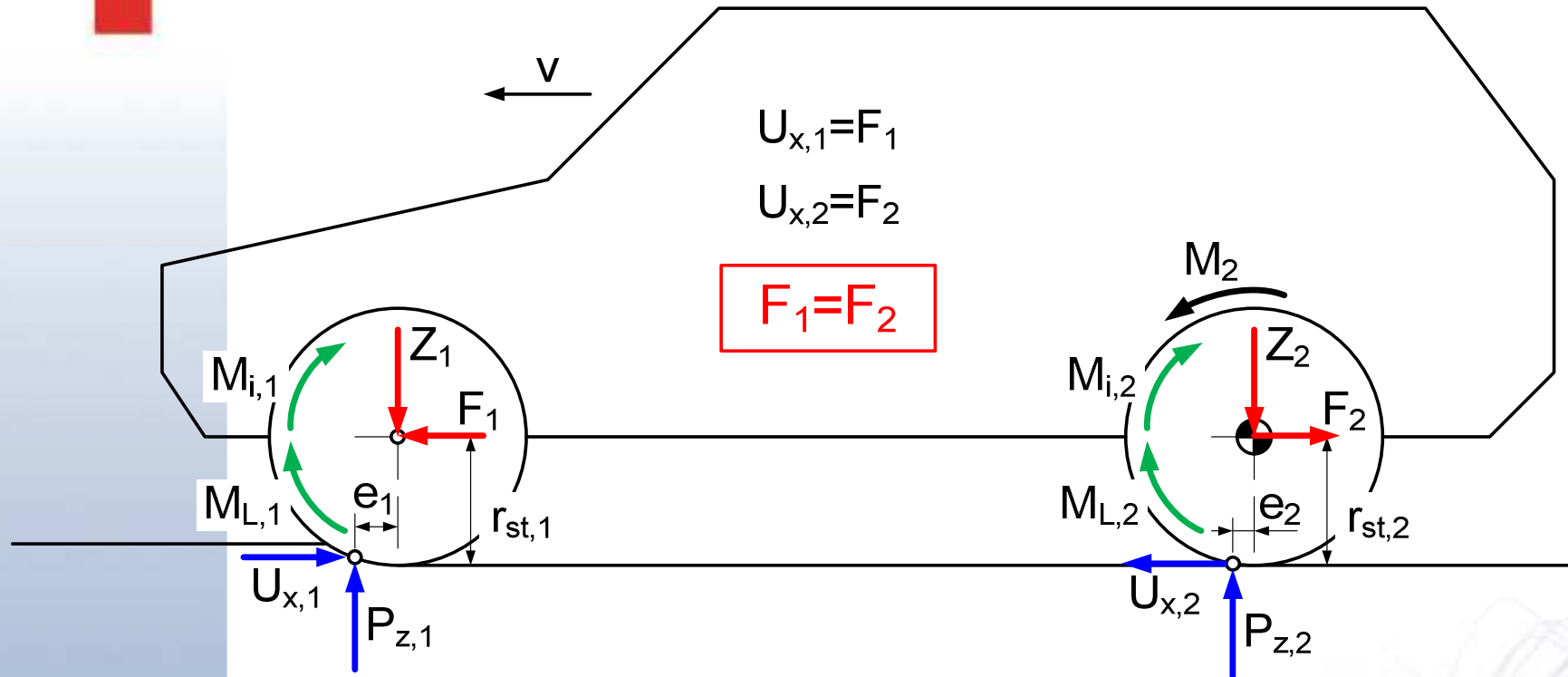

Pogonski navor za pogonsko silo /
Driving torque for a traction force:

$$\frac{M}{r_{st}} = \underbrace{Z \cdot \frac{e}{r_{st}}}_{R_f} + \frac{M_L}{r_{st}} + \frac{M_i}{r_{st}} + F; F = U_x$$

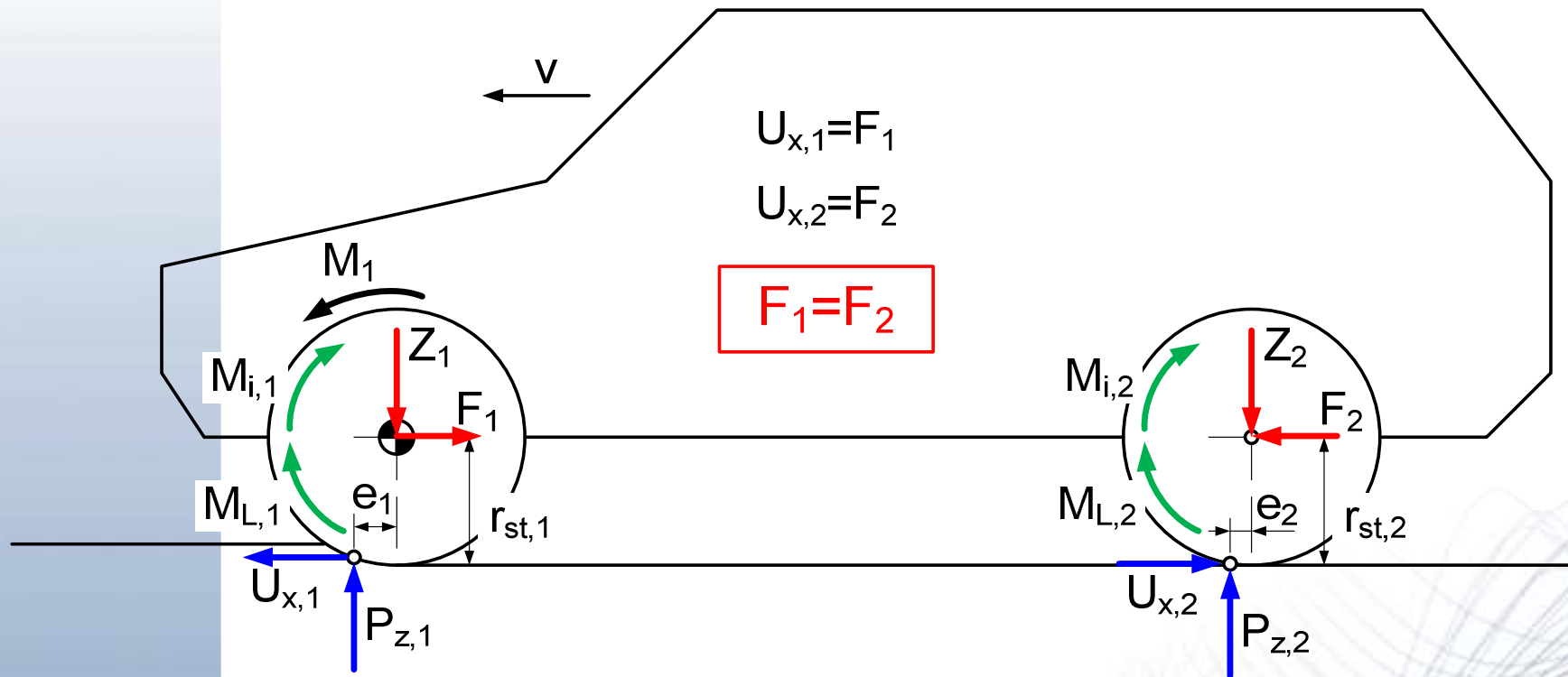# Kolesne obremenitve pri pogonu 4x2 zadaj / *Wheel loads for a rear 4x2 drive*

$U_{x,1}=F_1$

$U_{x,2}=F_2$

$$F_1=F_2$$

$$R_{f1} > R_{f2}$$

$$F_1 = F_2 > F_{tr,2} = \frac{Z_2}{\mu_{tr}} \Rightarrow zdrs / slip$$

Kolesne obremenitve pri pogonu 4x2 spredaj /
Wheel loads for a front 4x2 drive

Kolesne obremenitve pri pogonu 4x4 / *Wheel loads for a 4x4 drive*