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Thermal turbomachinery
Theoretical practice

Program: Erasmus

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## CONTENTS

1. PUMP HEAD AND PUMP POWER ................................................. 3

2. PERMISSIBLE PUMP SUCTION HEAD ....................................... 7

3. MULTIPLE STAGE COMPRESSION WITH INTERMEDIATE COOLING 10

4. LAVAL STEAM TURBINE STAGE (ZERO REACTION TURBINE STAGE) 14

5. STIRLING ENGINE ................................................................. 18

6. GAS TURBINE POWER PLANT ................................................. 22

7. COMBINED (GAS-STEAM) CYCLE ........................................... 26
1. PUMP HEAD AND PUMP POWER

For technological process 3 t/h water has to be pumped from the bottom reservoir, where the pressure is 1,8 bar, to the upper reservoir with atmospherically pressure of 1013 mbar. Water has the temperature of 40 °C. Hydraulic losses in suction pipe are in magnitude of 0,3 bar and in pressure pipe 0,4 bar. The internal efficiency of the pump is 0,85 and the mechanical 0,95. Overall electromotor efficiency is 0,8.

You have to determine following:

a) Absolute pressures before and after the pump
b) Pump head
c) The pressure distribution along the pipe system
d) Theoretical, internal and effective pump power and the power of electromotor for defined volume flow.

\[ \dot{m} = 3 \text{ t/h} = 0,833 \text{ kg/s} \]
\[ T = 40 \ ^\circ\text{C} \]
\[ p_1 = 1,8 \text{ bar} \]
\[ p_2 = 1013 \text{ mbar} = 1,013 \text{ bar} \]
\[ \Delta p_{up,s} = 0,3 \text{ bar} \]
\[ \Delta p_{up,t} = 0,4 \text{ bar} \]
\[ \eta_n = 0,85 \]
\[ \eta_m = 0,95 \]
\[ \eta_{em} = 0,8 \]
\[ d = 20 \text{ mm} = 0,02 \text{ m} \]

The origin is always the Bernoulli equation (\( \alpha \) is the starting point and \( \omega \) is the end point), which characterizes the mechanical energy of the fluid in specific point. The first expression in equation is the energy due to the pressure, the second due to the potential energy and the third due to the kinetic energy. On the right side of equation (at the point \( \omega \) the pressure losses, occurring between points \( \alpha \) in \( \omega \), are added.

\[ \frac{p_\alpha}{\rho \alpha g} + z_\alpha + \frac{c^2_\alpha}{2g} = \frac{p_\omega}{\rho \omega g} + z_\omega + \frac{c^2_\omega}{2g} + \frac{\Delta p_{up}}{\rho g} \]

With water the constant density can be assumed \((\rho_\alpha = \rho_\omega = \bar{\rho} = \rho)\).

a) The absolute pressure before the pump (at pump inlet)
At the suction part the Bernoulli equation between points 1 (bottom reservoir) and 1* (at pump inlet) can be written. From this equation we can express the pressure at the pump inlet.

\[ \frac{p_1}{\rho g} + z_1 + \frac{c^2_1}{2g} = \frac{p_1^*}{\rho g} + z_1^* + \frac{c^2_1^*}{2g} + \frac{\Delta p_{up,s}}{\rho g} \]
\[ p_{1*} = p_1 + \rho \ g (z_1 - z_{1*}) + \frac{\rho}{2} (c_1^2 - c_{1*}^2) - \Delta p_{up,s} \]

\[ \rho = 992 \text{ kg/m}^3 \]

With reservoirs we can always assume that the inflow and outflow speed of the water is so slow, that the water surface moving speed is neglected \((c_1 = c_2 = 0)\).

\[ c_{1*} = \frac{\dot{m}}{\rho \frac{\pi d^2}{4}} = \frac{0,833}{992 \frac{\pi \cdot 0,02^2}{4}} = 2,67 \text{ m/s} \]

\[ \dot{V} = c \cdot A \]

\[ \dot{m} = c \cdot \rho \cdot A \]

\[ c = \frac{\dot{m}}{\rho \cdot A} \]

\[ p_{1*} = 1,810^5 + 992 \cdot 9,81 (1 - 1,5) + \frac{992}{2} (0 - 2,67^2) - 0,3 \cdot 10^5 = 131867 \text{ Pa} = 1,32 \text{ bar} \]

At pump outlet, on the pressure side, we can write The Bernoulli equation between points 2* and 2. From this equation we can express the pump outlet pressure.

\[ \frac{p_{2*}}{\rho \ g} + \frac{c_{2*}^2}{2g} = \frac{p_2}{\rho \ g} + z_2 + \frac{c_2^2}{2g} + \frac{\Delta p_{up,t}}{\rho \ g} \]

\[ p_{2*} = p_2 + \rho \ g (z_2 - z_{2*}) + \frac{\rho}{2} (c_2^2 - c_{2*}^2) + \Delta p_{up,t} \]

\[ c_{1*} = c_{2*} = 2,67 \text{ m/s} \]

\[ p_{2*} = 1,013 \cdot 10^5 + 992 \cdot 9,81 \cdot 13,5 + \frac{992}{2} (0 - 2,67^2) + 0,4 \cdot 10^5 = 269140 \text{ Pa} = 2,69 \text{ bar} \]

b) Pump head

Pump head is pressure difference that has to be produced from the installed pump, written down in a different form. The pump has to produce the pressure difference that will push the specific amount of water from the starting point to the end point.

\[ H = \frac{p_{2*} - p_{1*}}{\rho \ g} \]

We insert the expressions for \(p_{1*}\) and \(p_{2*}\) and we get the general expression for pump head.

\[ H = \frac{p_2 - p_1}{\rho \ g} + (z_2 - z_1) + \frac{1}{2g} \left( \frac{c_2^2 - c_{1*}^2 + c_1^2 - c_{2*}^2}{\rho \ g} \right) + \frac{\Delta p_{up,s} + \Delta p_{up,t}}{\rho \ g} \]

The expressions of the equation characterize the energy differences between points 1 and 2, which have to be managed by a pump. Energy differences can be lower than zero, what means that energy difference (pressure, potential or due to speed conditions) between starting and end point contributes or “helps” the pump to suck and push the fluid within the pipeline. The expressions in the equation means:

1) The pressure difference between reservoirs
II Fluid level geodetic difference
III The kinetic energy difference of the water (= 0)
IV The pressure losses due to hydraulic and local losses

\[ H = \frac{(1,013 - 1,8) \times 10^5}{992.9,81} + 15 + 0 + \frac{(0,3 + 0,4) \times 10^5}{992.9,81} = 14,11\, \text{mVS} \]

\[ \frac{\rho g H}{1,0\, \text{bar}} = 2,7 \]
\[ 1,8 \]
\[ 1,3 \]
\[ 0,8,13,15 \]

\[ \frac{H}{\rho g H} = 2^* \]
\[ 1^* \]
\[ 1 \]
\[ 2 \]

From diagram is obvious why the pump have to increase the water pressure. In that way the pump give the fluid the energy, which helps the fluid passing between points 1 and 1* and 2* and 2.

c) The pressure distribution along the pipe system

d) Power

The theoretical power has to be added to the water mass flow to pass between points 1 and 2.

\[ P_t = \dot{m} g H = 0,833 \cdot 9,81 \cdot 14,11 = 115,3\, \text{W} \]

Because the compression within the pump doesn‘t run ideally (isentropically), the pump needs the internal power to overcome internal losses due to irreversibility’s. The internal power is because of the internal efficiency larger than the theoretical power.

\[ P_n = \frac{P_t}{\eta_n} = \frac{115,3}{0,85} = 135,7\, \text{W} \]

One part of the power within the pump is used to overcome frictional losses within pump bearings. Because of that the pump effective power is needed, which is larger than internal power because of the mechanical efficiency.

\[ P_e = \frac{P_n}{\eta_m} = \frac{135,7}{0,95} = 142,8\, \text{W} \]
In addition to run the pump, electromotor has to overcome its own losses (electrical and mechanical) and therefore consuming electrical power, which is even larger than effective power of the pump due to the overall electromotor efficiency.

\[ P_{em} = \frac{P_e}{\eta_{em}} = \frac{142,8}{0,8} = 178,5 \text{ W} \]
2. PERMISSIBLE PUMP SUCTION HEAD

The lake level is varying for 3 m in average in one year. The pump station is set 5 m above upper lake level. The suction pipe has the length of 9 m, diameter of 70 mm and total local losses coefficient $\Sigma \zeta = 2.1$. The average pressure during one year is 981 mbar and the average water temperature is 15 °C.

1. Calculate the pump inlet pressure if the water mass flow is 14 kg/s and:
   a) upper lake level (lake level 5 m below the pump),
   b) lower lake level (lake level 8 m below the pump) and
   c) middle lake level (lake level 6,5 m below the pump).

2. Determine the lake level at which is the limit for pumping of the 14 kg/s of water (permissible pump suction head).

3. Sketch the diagram of dependence between permissible pump head and fluid mass flow.

1. **Pump inlet pressure**

   The Bernoulli equation has to be set between lake level and pump inlet:

   $$ \frac{p_0}{\rho g} + \frac{c_0^2}{2g} = \frac{p_1}{\rho g} + z_1 + \frac{c_1^2}{2g} + \frac{\Delta p_{up}}{\rho g} $$

   $$ p_1 = p_0 + \rho g (z_0 - z_1) + \frac{\rho}{2} \left( c_0^2 - c_1^2 \right) - \Delta p_{up} $$

   In all cases it is

   $$ \rho(15 \, ^\circ \text{C}) = 999,1 \, \text{kg/m}^3 $$

   $$ c_0 = 0 $$

   $$ c_1 = \frac{4m}{\pi d^2 \rho} = \frac{4 \cdot 14}{\pi \cdot 0.07^2 \cdot 999,1} = 3,64 \, \text{m/s} $$

   $$ \Delta p_{up} = \Delta p_{up,c} + \Delta p_{up,d} = \sum c \rho \frac{c^2}{2} + \frac{L}{d} \lambda \frac{c^2}{2} $$

   $$ \lambda = 0,31 \left( \log(0.143\text{Re}) \right)^2 $$

   (Colebrook equation)

   $$ \text{Re} = \frac{c d}{\nu} = \frac{3,64 \cdot 0,07}{10^{-6}} = 254877 $$

   $$ \lambda = 0,0149 $$

   $$ \Delta p_{up} = 2,1 \frac{999,1 \cdot 3,64^2}{2} + \frac{9}{0,07} \cdot 0,0149 \cdot 999,1 \cdot 3,64^2 = 26593 \, \text{Pa} $$
The only difference in cases a), b) and c) is just height. With variable height the inlet pump pressure vary too.

a) \((z_0 - z_1)_a = 5\) m

\[
p_{1a} = 0.981 \cdot 10^5 + 999.1 \cdot 9.81 \cdot 5 + \frac{999.1}{2} (0 - 3.64^2) - 26593 = 15878 \text{ Pa} = 0.159 \text{ bar}
\]

b) \((z_0 - z_1)_b = 8\) m

\[
p_{1b} = 0.981 \cdot 10^5 + 999.1 \cdot 9.81 \cdot 8 + \frac{999.1}{2} (0 - 3.64^2) - 26593 = -13526 \text{ Pa}
\]

The pressure in any case couldn’t be less than 0, therefore is not possible to suck specific mass flow in the existing system on the specific height. With lower mass flow we get lower expression for speed and lower expression for pressure so the pressure at the pump inlet is set to normal value (suction on the specific height is possible at mass flow 10 kg/s)

c) \((z_0 - z_1)_c = 6.5\) m

\[
p_{1c} = 0.981 \cdot 10^5 + 999.1 \cdot 9.81 \cdot 6.5 + \frac{999.1}{2} (0 - 3.64^2) - 26593 = 1176 \text{ Pa} = 0.012 \text{ bar}
\]

The pressure in this case is positive, but the suction at given height is still not possible. Saturation temperature at pressure 0.012 bar is 9.35 °C what means that the water with temperature 15 °C starting to boil even sooner. The possible solution in this case is lower the mass flow.

2. **Permissible pump suction head:**

The limitation for suction head is saturation pressure at given water temperature, because the water in suction side shouldn’t boil. In limit case:

\[
\frac{p_0}{\rho g} + z_0 + \frac{c_0^2}{2g} = \frac{p_s}{\rho g} + z_{mej} + \frac{c^2}{2g} + \frac{\Delta p_{up}}{\rho g}
\]

Pressure \(p_s\) is saturation pressure at given water temperature. **The permissible suction head has to be lower than height difference** \(z_{mej} - z_0\).

\[
p_1 < p_s(T_s)
\]

\[
H_{s,up} = \frac{p_0}{\rho g} - \frac{p_s}{\rho g} - \frac{c^2}{2g} - \frac{\Delta p_{up}}{\rho g}
\]

\(p_s(15 \degree C) = 0.01706 \text{ bar} = 1706 \text{ Pa}

\[
H_{s,up} < \frac{98100}{999,1 \cdot 9.81} - \frac{1706}{999,1 \cdot 9.81} - \frac{3.64^2}{2 \cdot 999,1} - \frac{26593}{999,1 \cdot 9.81} = 6.44 \text{ m}
\]
3. The permissible suction head diagram
3. **MUTIPLE STAGE COMPRESSION WITH INTERMEDIATE COOLING**

The air mass flow of 1,5 kg/s with temperature of 25 °C and pressure 0,7 bar is compressed to 16 bar. Because of the limitation of the air temperature after each compression that is 150 °C, the compression has to be done in multiple stage compressor with intermediate cooling. Politropical exponent of each compression stage is \( n = 1.45 \). Calculate:

a) The air temperature after compression if the compression to 16 bar is done in single stage compressor;

b) Stages required for compressing the air with temperature limitation;

c) Draw the sketch of the facility.

d) The heat flow transferred from air.

e) Draw the compression process in \( h – s \) diagram for air.

Multiple compression is used in cases where the temperature after one stage compression is above the temperature that is allowed in the system, or, where desired pressure after one stage compression is not to be reached.

**A) TEMPERATURE OF THE AIR WHEN ONLY ONE STAGE COMPRESSION IS APPLIED**

From gas laws equation \( p v = R T \) and politropical equation \( p v^n = \text{const} \). The link between temperature and pressure with poltrope compression can be written.

\[
\frac{T_k}{T_z} = \left( \frac{p_k}{p_z} \right)^{\frac{n-1}{n}} = e^{\frac{n-1}{n}}
\]

\( k \) – state before compression

\( z \) – state after compression

\( e \) – compression ratio

\( n \) – politropical exponent

Politrope equation:

\[
p_1 v_1^n = p_2 v_2^n \rightarrow p_1 p_2 = \left( \frac{v_2}{v_1} \right)^n
\]

Gas laws equation:

\[
\frac{mRT}{v_1} = \left( \frac{v_2}{v_1} \right)^n
\]

\[
\frac{mRT}{v_2} = \left( \frac{v_2}{v_1} \right)^n
\]

\[
\frac{T_1}{T_2} v_2 = \left( \frac{v_2}{v_1} \right)^n \rightarrow \frac{T_1}{T_2} = \left( \frac{v_2}{v_1} \right)^{n-1}
\]

with consideration

\[
\frac{v_2}{v_1} = \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}}
\]

\[
\frac{T_1}{T_2} = \left( \frac{p_1}{p_2} \right)^{\frac{n-1}{n}}
\]
With one stage compression the temperature after compression can be calculated on the basis of known data.

\[ T_{k,a} = T_0 \left( \frac{p_k}{p_0} \right)^{\frac{n-1}{n}} = 298 \left( \frac{16}{0.7} \right)^{\frac{1.45-1}{1.45}} = 787 \text{ K} = 514 \text{ °C} \]

**b) The number of stages required and pressure after every compression stage**

If the temperature is limited after every compression stage, the largest compression ratio, where the temperature limit won’t be exceeded, can be calculated with the same equation.

\[ \varepsilon_{dop} = \left( \frac{T_{dop}}{T_0} \right)^{\frac{n-1}{n}} = \left( \frac{423}{298} \right)^{\frac{1.45}{1.45-1}} = 3.09 \]

With permissible compression ratio the limit temperature is reached after every compression stage, but the pressure after last compression stage won’t be 16 bar. Actual compression ratio can be lower than permissible compression ratio. Before we calculate actual compression ratio, we have to determine the actual number of stages.

After \( x \) compression stages the pressure is: \( p_x = p_0 \varepsilon^x \)

For known compression ratio \( \varepsilon \) we determine the required number of stages with: \( x = \frac{\ln p_k}{\ln \varepsilon} \)

With permissible compression ratio \( \varepsilon_{dop} \) the required ‘permissible’ number of stages is

\[ x_{dop} = \frac{\ln p_k}{\ln \varepsilon_{dop}} = \frac{\ln 16}{\ln 3.09} = 2.77 \]

Actual number of stages has to be integer number, that’s why we round up the number \( x_{dop} \) to first larger integer number. With that is actual compression number lower then permissible one and actual temperature after each compression stage is lower than maximal permissible (150 °C) temperature.

\( x_{dop} = 2.77 \Rightarrow x = 3 \)

We calculate the actual compression ratio within single stage. We consider that compression ratios are equal in all compression stages.
\[ \varepsilon = \left( \frac{p_k}{p_0} \right)^{\frac{1}{s}} = 2.84 \]

**Pressures after each compression stage:**

\[ p_1 = p_0 \varepsilon = 0.7 \cdot 2.84 = 1.987 \text{ bar} \]
\[ p_2 = p_1 \varepsilon = p_0 \varepsilon^2 = 0.7 \cdot (2.84)^2 = 5.638 \text{ bar} \]
\[ p_3 = p_2 \varepsilon = p_0 \varepsilon^3 = 0.7 \cdot (2.84)^3 = 16 \text{ bar} = p_k \]

The pressure after last compression stage has to be equal to desired pressure \( p_k \) that is 16 bar.

The actual temperature after each compression stage is

\[ T_k = T_0 \varepsilon^n = 298 \cdot 2.84^{\frac{1.45-1}{1.45}} = 412 \text{ K} = 139 \text{ °C} \]

c) **The facility sketch**

With known number of compression stages we can draw the sketch of the compression facility.

d) **The transferred heat from air**

The heat is transferred from air after first and second compression stage between points 1 and 1' and 2 and 2' (see sketch above).

\[ \dot{Q}_{sr} = m_{sr} (h_{sr1} - h_{sr1'} + h_{sr2} - h_{sr2'}) \]

\[ h_{sr1} = h(p_1, T_k) = 140.7 \text{ kJ/kg} \]
\[ h_{sr1'} = h(p_1, T_0) = 25.1 \text{ kJ/kg} \]
\[ h_{sr2} = h(p_2, T_k) = 140.7 \text{ kJ/kg} \]
\[ h_{sr2'} = h(p_2, T_0) = 25.1 \text{ kJ/kg} \]

\[ \dot{Q}_{sr} = 1.5 \cdot (140.7 - 25.1 + 140.7 - 25.1) = 346.8 \text{ kW} \]

We read the values from table or \( h - s \) diagram for air!
e) The compression process in $h – s$ diagram for air
4. LAVAL STEAM TURBINE STAGE (ZERO REACTION TURBINE STAGE)

The steam of 200 °C and 4,0 bar enters Laval steam turbine stage with the speed of 50 m/s. The pressure on the stage outlet is 1,6 bar. The middle diameter of the stage which is rotating with 3000 min\(^{-1}\) is 890 mm. The rest of the data for the Laval stage: \(\alpha_1 = 20^\circ\); \(\beta_2 = \beta_1 - 2^\circ\); \(\psi_1 = 0,96\); \(\psi_2 = 0,9\). Calculate
a) speed characteristics in the stage (relative and absolute speed on the stage inlet and outlet),
b) velocity triangles (draw them) and angle \(\alpha_2\),
c) the isentropic efficiency,
d) the stage power with steam mass flow of 0,5 kg/s.

The turbine stage is constituted of one stator and one rotor.

The zero reaction stage is stage in which are the pressures in points 1 and 3 (before and after rotor) equal. The whole expansion process take place already in the stator and internal energy of the steam is converted into kinetic energy already in the stator. The speed of the steam increases.
a, b) Speed characteristics in the stage and velocity triangles:
The absolute and relative speed on the turbine inlet and outlet should be calculated. The
basement is energy balance, where we should take into account the speed of the steam (kinetic
energy). The total energy of the fluid is expressed with total enthalpy.

\[ h_0^* = h_0 + \frac{c_0^2}{2} \]

theoretical total enthalpy at stator outlet \( h_1^* = h_{1th} + \frac{c_{1th}^2}{2} \)

Because the expansion process in the stator is relative quick, we can assume that total enthalpy at
the stator inlet and outlet is the same. With that presumption we can express theoretical outlet
steam speed. (the state \( 0 \) we determine with the help of tables of thermodynamic properties of
water and steam; the state \( 1th \) falls into two-faze area, first you determine \( x \), than \( h_{1th} \). in the
equation the enthalpy has to be in J/kg; the speed at the inlet is always much lower than the
speed of the steam later).

\[ c_{1th} = \sqrt{2(h_0 - h_{1th}) + c_0^2} \]

In the point \( 1th \) the \( p_{1th} \) and \( s_{1th} \) are known, we want to determine the enthalpy:

\[ x_{1th} = \frac{s_{1th} - s'(p_{1th})}{s''(p_{1th}) - s'(p_{2th})} \]

\[ h_{1th} = h'(p_{1th}) + x_{1th}(h''(p_{1th}) - h'(p_{1th})) \]

\[ h_{1th} = 2685,1 \text{ kJ/kg} \]

\[ c_{1th} = \sqrt{2(2860,4 - 2685,1) \cdot 10^3 + 50^2} = 594,2 \text{ m/s} \]

The actual speed is lower because of the friction losses in stator that are included in loss
coefficient in the stator, \( \psi_1 \).

\[ c_1 = c_{1th} \psi_1 = 594.2 \cdot 0.96 = 570.4 \text{ m/s} \]

At the rotor inlet the magnitude of the absolute speed \( (c_1) \) and absolute speed direction \( (\alpha_1) \)
are known. We can calculate circumference speed and draw the velocity triangle in the point 1.

\[ u = \frac{D \omega}{2} = \frac{\pi D n}{60} = \frac{\pi \cdot 0.89 \cdot 3000}{60} = 139.8 \text{ m/s} \]

From geometry it can be calculated:

\[ w_1 = \sqrt{w_{1u}^2 + w_{1a}^2} \]

\[ w_{1u} = c_{1u} - u = c_1 \cos \alpha_1 - u = 570.4 \cdot \cos 20^\circ - 139.8 = 396.2 \text{ m/s} \]
\( w_{1u} = c_{1u} = c_1 \sin \alpha_1 = 570,4 \cdot \sin 20^\circ = 195,09 \text{ m/s} \)
\[ w_1 = 441,63 \text{ m/s} \]
\[ \beta_1 = \arccos \left( \frac{w_{1u}}{w_1} \right) = \arccos \left( \frac{396,2}{441,63} \right) = 26,21^\circ \]

Theoretically the relative velocity at the rotor outlet and the relative velocity at the rotor inlet are equal, but actually the relative velocity is a bit lower because of the friction losses in the rotor.

\[ w_2 = w_1 \psi_2 = 441,6-0,9 = 397,4 \text{ m/s} \]

The circumference speed at rotor outlet is equal as at the rotor inlet, because the middle diameter of the stage is not changed. We should calculate the missing data for the velocity triangle in the point 2.

From geometry can be calculated:
\[ c_2 = \sqrt{c_{2u}^2 + c_{2a}^2} \]
\[ c_{2u} = w_{2u} = w_2 \sin \beta_2 \]
\[ c_{2a} = w_{2a} - u = w_2 \cos \beta_2 - u \]
\[ \beta_2 = \beta_1 - 2^\circ = 24,21^\circ \text{ (in the exercise definition)} \]
\[ c_{2a} = 397,4 \cdot \sin 24,21^\circ = 162,97 \text{ m/s} \]
\[ c_{2a} = 397,4 \cdot \cos 24,21 - 139,8 = 222,65 \text{ m/s} \]
\[ c_2 = 278,9 \text{ m/s} \]
\[ \alpha_2 = \arctan \left( \frac{c_{2a}}{c_{2u}} \right) = \arctan \left( \frac{162,97}{222,65} \right) = 36,2^\circ \]

**c) The isentropic efficiency:**
\[ \eta_t = \frac{P_{dej}}{P_{th}} = \frac{h_3 - h_1}{h_0 - h_{1th}} \]

In this equation we don’t know the enthalpy in the point 3 (see expansion process in h-s diagram). This enthalpy can be calculated from theoretical enthalpy after expansion (point 1th) that the losses in stator \( z_1 \) and rotor \( z_2 \) and outlet stage losses \( z_3 \) are added.
\[ h_3 = h_{1th} + (z_1 + z_2 + z_3) \]
\[
\begin{align*}
    z_1 &= \frac{c_{1h}^2}{2} - \frac{c_1^2}{2} = \frac{c_{1h}^2}{2} \left(1 - \psi_1^2\right) = \frac{594.2^2}{2} \left(1 - 0.96^2\right) = 13840 \text{ J/kg} \\
    z_2 &= \frac{w_1^2}{2} - \frac{w_2^2}{2} = \frac{w_1^2}{2} \left(1 - \psi_2^2\right) = \frac{441.6^2}{2} \left(1 - 0.9^2\right) = 18526 \text{ J/kg} \\
    z_3 &= \frac{c_2^2}{2} = \frac{275.9^2}{2} = 38060 \text{ J/kg}
\end{align*}
\]

The outlet stage loss is the loss only with last turbine stage. At all other stages the outlet rotor steam speed plays part of the total steam enthalpy at the next turbine stage inlet. Because of that the kinetic energy is not lost.

\[
h_3 = 2685.1 + (13,840 + 18,526 + 38,060) = 2752.5 \text{ kJ/kg}
\]

\[
\eta_t = \frac{2860.4 - 2752.5}{2860.4 - 2685.1} = 0.617
\]

d) Stage power:

\[
P_t = m \left(h_0 - h_3\right) = \dot{m} \left(h_0 - h_{1h}\right) \eta_t = 0.5 \cdot (2860.4 - 2752.5) = 53.95 \text{ kW}
\]

**Analytical determination of the velocity triangles**

*known: \(u, c_1, \alpha_1\)*

the circumferential component of the relative inlet speed: \(w_{1u} = c_{1u} - u = c_1 \cos \alpha_1 - u\)

the axial component of the relative inlet speed: \(w_{1a} = c_{1a} = c_1 \sin \alpha_1\)

relative inlet velocity: \(w_1 = \sqrt{w_{1u}^2 + w_{1a}^2}\)

the direction of the relative inlet velocity: \(\beta_1 = \arccos \frac{w_{1u}}{w_1}\)

the circumferential component of the absolute outlet speed: \(c_{2u} = w_{2u} - u = w_2 \cos \beta_2 - u \) (angle \(\beta_2\) is known from the exercise definition regarding to angle \(\beta_1\))

axial component of the absolute outlet speed: \(c_{2a} = w_{2a} = w_2 \sin \beta_2\)

absolute outlet speed: \(c_2 = \sqrt{c_{2u}^2 + c_{2a}^2}\)

the direction of the absolute outlet speed: \(\alpha_2 = \arccos \frac{c_{2u}}{c_2}\)
5. **STIRLING ENGINE**

Inside Stirling engine the right cycle occurs. With Stirling engine we get the mechanical work. The media inside the engine is air, that is treated like the ideal gas with the gas constant of $R = 287 \, J/kgK$ and politropical exponent of $\kappa = 1,4$. The minimal pressure inside the process is 1,2 bar and the maximal pressure is 22 bar. The Stirling engine works between temperatures 290 K and 700 K. Regarding the geometry of the cylinder and his working volume, we define the air mass flow inside Stirling engine.

a) Calculate the heat flow in and out of the process.

b) Calculate the mechanical work of the process.

c) Calculate the efficiency of the Stirling cycle.

d) What is the efficiency with the regeneration degree of $\sigma = 0,8$ ?

e) What is the heat flow into the process with the heat regeneration? Calculate the heat flow reduction in % !

$$R = 287 \, J/kgK$$

$$\kappa = 1,4$$

$$p_{\text{min}} = p_1 = 1,2 \, \text{bar}$$

$$p_{\text{max}} = p_3 = 22 \, \text{bar}$$

$$T_{\text{min}} = T_1 = T_2 = 290 \, \text{K}$$

$$T_{\text{max}} = T_3 = T_4 = 700 \, \text{K}$$

$$\dot{m} = 6 \, \text{kg/s}$$

The Stirling process is the cycle that runs within two isotherms and two isochors. The “lower” isochor represents the simultaneous compression and heat transport out of the process and the “upper” isochor simultaneous expansion and heat transport in to the process.
a) Heat flow in and out of the Stirling cycle

Heat flow into the Stirling cycle is compounded from heat that should be transferred into the process within the isochor (from 2 to 3) and the heat transferred into the process within the isotherm (from 3 to 4).

\[ \dot{Q}_{\text{do}} = \dot{Q}_{23} + \dot{Q}_{34} \]

The heat between point 2 and 3 is transferred into the process within isochor (at a constant volume). For air is isobaric specific heat \( c_p = 1,005 \text{ kJ/kgK} \). Isochoric and isobaric heat are connected with the term: \( \kappa = c_p / c_v \)

\[ \dot{Q}_{23} = m c_v \left( T_{\text{max}} - T_{\text{min}} \right) = 6 \cdot \frac{1,005}{1,4} (700 - 290) = 1765,93 \text{ kW} \]

Between points 3 and 4 the isothermal heat transfer into the process occurs:

\[ \dot{Q}_{34} = m T_{\text{max}} \left( s_4 - s_3 \right) = m T_{\text{max}} \left( c_p \ln \frac{T_4}{T_3} - R \ln \frac{p_4}{p_3} \right) \]

The pressure in the point 4 can be calculated with the help of isochoric transformation between points 4 and 1:

\[ \frac{p_4 V_4}{T_4} = \frac{p_1 V_1}{T_1} \quad \Rightarrow \quad p_4 = p_1 \frac{T_4}{T_1} = p_{\text{min}} T_{\text{max}} = 1,2 \cdot \frac{700}{290} = 2,90 \text{ bar} \]

\[ \dot{Q}_{34} = m T_{\text{max}} \left( c_p \ln \frac{T_{\text{max}}}{T_{\text{min}}} - R \ln \frac{p_{\text{min}} T_{\text{max}}}{p_{\text{max}} T_{\text{min}}} \right) = m T_{\text{max}} R \ln \frac{p_{\text{max}} T_{\text{min}}}{p_{\text{min}} T_{\text{max}}} \]

\[ \dot{Q}_{34} = 6 \cdot 700 \cdot 287 \cdot \ln \frac{22 \cdot 290}{1,2 \cdot 700} = 2443,97 \text{ kW} \]

\( \dot{Q}_{\text{do}} = 4209,9 \text{ kW} \)
The heat transferred from the process is compounded from heat transferred within the isotherm (from 1 to 2) and the heat transferred within isochor (from 4 to 1).

\[ \dot{Q}_{od} = \dot{Q}_{41} + \dot{Q}_{12} \]

\[ \dot{Q}_{41} = \dot{m} c_v (T_{\min} - T_{\max}) = 6 \cdot 1.005 \cdot \left( \frac{1005}{1.4} \right) \cdot (290 - 700) = -1765.93 \text{ kW} \]

\[ \dot{Q}_{12} = \dot{m} T_{\min} (s_2 - s_1) = \dot{m} T_{\min} R \ln \left( \frac{P_{\min} T_{\max}}{P_{\max} T_{\min}} \right) = 6 \cdot 290 \cdot 287 \cdot \ln \left( \frac{1.2 \cdot 700}{22 \cdot 290} \right) = -1012.5 \text{ kW} \]

\[ \dot{Q}_{od} = -2778.43 \text{ kW} \]

The heat transferred from the process has the negative sign (that’s correct), but we will left the sign out in next equations.

**b) The mechanical power**

\[ P = \dot{Q}_{do} - \dot{Q}_{od} = 4209.9 - 2778.43 = 1431.47 \text{ kW} \]

**c) The efficiency of the Stirling cycle**

\[ \eta = \frac{P}{\dot{Q}_{do}} = \frac{1431.47}{4209.9} = 0.340 \]

**d) The efficiency of the Stirling cycle with heat regeneration**

The efficiency is relative low. We can lift the efficiency with reduction the heat transferred into the process. The heat within isochor \( \dot{Q}_{41} \) is transferred out of the process from 700 K, on the other side the heat is transferred into the process already from temperature 290 K to 700 K. For that reason for the efficiency enlargement in the Stirling engine the heat regeneration is used. That means that the part of the heat that is transferred out of the process, \( \dot{Q}_{41} \), is transferred to the heat that is transferred into the process \( \dot{Q}_{34} \). Theoretically heat flows are equal, but in reality the all heat can not be transferred from 4-1 to 3-4, because for the heat transport the temperature difference has to exist. On the one hand the air could not be cooled to the \( T_{\min} \), on the other hand with the heat regeneration \( T_{\max} \) could not be reached. The regeneration degree \( \sigma \) is the indicator what part of temperature difference we were able to use for heat regeneration. On the diagram showed above that means that heat is actually transferred into the process from 2' to 3 and transferred out of the process from 4' do 1. All other heat is obtained with regeneration.

\[ \sigma = \frac{T_2 - T_{\min}}{T_{\max} - T_{\min}} = \frac{T_{\max} - T_4'}{T_{\max} - T_{\min}} \]

\[ T_2' = T_{\min} + \sigma (T_{\max} - T_{\min}) = 290 + 0.8 \cdot (700 - 290) = 618 \text{ K} \]

\[ T_4' = T_{\max} - \sigma (T_{\max} - T_{\min}) = 700 - 0.8 \cdot (700 - 290) = 372 \text{ K} \]
Due to heat regeneration the heat flow required into the process is reduced, but the power remains the same. That’s why the efficiency is higher.

\[ \dot{Q}_{do,R} = \dot{Q}_{23} + \dot{Q}_{34} = \dot{m}c_v(T_{\text{max}} - T_2) + \dot{Q}_{34} = 6 \cdot \frac{1.005}{1.4} (700 - 618) + 2443.97 = 2797.16 \text{ kW} \]

\[ \eta = \frac{1431.48}{2797.16} = 0.512 \]

e) The reduction of the heat transported into the process due to heat regeneration

In the case of heat regeneration the heat transferred into the process is 2797.16 kW. So the heat transported into the process is lower. The difference is regenerated heat:

\[ \Delta \dot{Q}_{do} = \dot{Q}_{do} - \dot{Q}_{do,R} = 4209.9 - 2797.16 = 1412.74 \text{ kW} \]

The share of the heat that is regenerated is:

\[ \frac{\Delta \dot{Q}_{do}}{\dot{Q}_{do}} = \frac{1412.74}{4209.9} = 0.336 \Rightarrow 33.6 \% \]

Because of the heat regeneration is the heat flow that has to be transferred into the process lower. Almost 34 % of the heat transferred out of the process is regenerated and that affects the efficiency.
6. GAS TURBINE POWER PLANT

Gas construction operates by followed parameters of air and gas:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sign</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass flow of air</td>
<td>( \dot{m}_{\text{air}} )</td>
<td>28 kg/s</td>
</tr>
<tr>
<td>Air pressure before compression (atmospheric)</td>
<td>( p_0 )</td>
<td>1 bar</td>
</tr>
<tr>
<td>Air temperature before compression (atmospheric)</td>
<td>( T_0 )</td>
<td>20 °C</td>
</tr>
<tr>
<td>Air pressure after compression</td>
<td>( p_1 )</td>
<td>10 bar</td>
</tr>
<tr>
<td>Air temperature after compression</td>
<td>( T_1 )</td>
<td>400 °C</td>
</tr>
<tr>
<td>Gas temperature before turbine</td>
<td>( T_3 )</td>
<td>1100 °C</td>
</tr>
<tr>
<td>Gas temperature after turbine</td>
<td>( T_4 )</td>
<td>550 °C</td>
</tr>
</tbody>
</table>

![Diagrams](image)

**a) Simple gas cycle**

**b) Simple gas cycle with regeneration**

**c) Combined cycle**

Determine:

a) Internal efficiency of compressor and turbine, power of turbine and thermodynamic efficiency of simple gas cycle;

b) The heat saving, thermodynamic efficiency of the cycle and the outlet gas temperature in the case of regenerative heating \( \sigma = 0.83 \) (the level of regeneration);

c) The heat saving and overall efficiency of combined cycle if the outlet gases transmit heat to steam cycle and with that the gases are cooled to the temperature \( T_6 = 180 \) °C; the thermodynamic efficiency of steam cycle is \( \eta_{\text{PA}} = 0.4 \);

d) Compare heat savings, temperatures of exhaust gases and the thermodynamic efficiencies of all three examples.

By analysis of gas process neglect pressure drops and mass flow of fuel. For exhaust gases consider thermodynamic properties of dry air (\( h - s \) diagram).
a) **Simple gas cycle**

**Internal compressor efficiency:**

\[ \eta_k = \frac{P_{k,sh}}{P_k} = \frac{\dot{m}_{zr}(h_{z,k} - h_0)}{\dot{m}_{zr}(h_1 - h_0)} = \frac{h_{z,k} - h_0}{h_1 - h_0} \]

\[ h_0 = h(p_0, T_0) = 19.93 \text{ kJ/kg} \]

\[ h_1 = h(p_1, T_1) = 411.52 \text{ kJ/kg} \]

\[ h_{z,k} = h(p_1, s_0) \]

\[ s_0 = s(p_0, T_0) \]

\[ h_{z,k} = 293.74 \text{ kJ/kg} \]

\[ \eta_k = \frac{293.74 - 19.93}{411.52 - 19.93} = 0.699 \]

**Internal turbine efficiency:**

\[ \eta_t = \frac{P_{t,sh}}{P_t} = \frac{\dot{m}_{zr}(h_{z,t} - h_4)}{\dot{m}_{zr}(h_3 - h_{z,k})} = h_3 - h_4 \]

\[ h_3 = h(p_3, T_3) = 1211.01 \text{ kJ/kg}, \quad p_3 = p_1 = 10 \text{ bar} \]

\[ h_4 = h(p_4, T_4) = 574.42 \text{ kJ/kg}, \quad p_4 = p_0 = 1 \text{ bar} \]

\[ h_{z,t} = h(p_4, s_3) \]

\[ s_3 = s(p_3, T_3) \]

\[ h_{z,t} = 517.40 \text{ kJ/kg} \]

\[ \eta_t = \frac{1211.01 - 574.42}{1211.01 - 517.40} = 0.918 \]

**Internal compressor power:**

\[ P_k = \dot{m}_{zr}(h_1 - h_0) = 28(411.52 - 19.93) = 10964 \text{ kW} \]

**Internal turbine power:**

\[ P_t = \dot{m}_{zr}(h_3 - h_4) = 28(1211.01 - 574.42) = 17825 \text{ kW} \]

**Net power of turbine:**

\[ P_p = P_t - P_k = 17825 - 10964 = 6861 \text{ kW} \]

**Inlet heat flux:**

\[ \dot{Q}_{do} = \dot{m}_{zr}(h_3 - h_1) = 28(1211.01 - 411.52) = 22386 \text{ kW} \]
Cycle efficiency:
\[ \eta_p = \frac{P_p}{Q_{do}} = \frac{6861}{22386} = 0.31 \]

b) Heat regeneration
Exiting exhaust gases (mostly air) from the gas process have high temperature \((T_4 = 550 \, ^\circ C)\). With the heat of exhaust gases we can heat compressed air and that reduces heat flux delivered by fuel. The highest quantity of regenerated heat is determined by difference between exhaust gas temperature \(T_4\) and air temperature after compressor \(T_1\). Real regenerated heat stream is a bit smaller. The level of regeneration is defined by equation:
\[ \sigma = \frac{\dot{Q}_r}{Q_{r,\text{max}}} = \frac{T_2 - T_1}{T_4 - T_1} = \frac{T_4 - T_5}{T_4 - T_1} \]

When the level of regeneration is known, we can calculate compressed air temperature at heat regenerator outlet:
\[ T_2 = T_1 + \sigma(T_4 - T_1) = 400 + 0.83(550 - 400) = 524.5 \, ^\circ C \]

Regenerated heat flux:
\[ \dot{Q}_r = m_{sr}(h_2 - h_1) = m_{sr}(h_4 - h_5) \]
\[ h_2 = h(p_2, T_2) = 546.62 \, \text{kJ/kg} \]
\[ \dot{Q}_r = 28(546.62 - 411.52) = 3783 \, \text{kW} \]

Exhaust gas temperature at heat regenerator outlet:
\[ T_5 = T_4 - \sigma(T_4 - T_1) = 550 - 0.83(550 - 400) = 425.5 \, ^\circ C \]
This temperature is still high but it cannot be reduced more with heat regenerator (limit is \(T_1 = 400 \, ^\circ C\)).

Reduced inlet heat flux:
\[ \dot{Q}_{do,r} = \dot{Q}_{do} - \dot{Q}_r = 22386 - 3783 = 18603 \, \text{kW} \]

Efficiency of gas cycle with regeneration:
\[ \eta_{p,r} = \frac{P_p}{\dot{Q}_{do,r}} = \frac{6861}{18603} = 0.37 \]
c) **Combined cycle**

*Instead of using the heat of exhaust gas in heat regenerator it can be used for steam production in utilizator (steam boiler).*

Heat power of utilizator:
\[
\dot{Q}_u = m_{\text{zr}} (h_4 - h_6)
\]
\[
h_6 = h(p_6, T_6) = 181,85 \text{ kJ/kg}; \, p_4 = p_6 = p_0 = 1 \text{ bar}; \, T_6 = 180 ^\circ C
\]
\[
\dot{Q}_u = 28 (411,52 - 181,85) = 10992 \text{ kW}
\]

Internal steam turbine power:
\[
\eta_{pa} = \frac{P_p}{\dot{Q}_a} \Rightarrow P_p = \eta_{pa} \cdot \dot{Q}_u = 0,4 \cdot 10992 = 4397 \text{ kW}
\]

Efficiency of combined cycle:
\[
\eta_{pk} = \frac{P_p + P_{pa}}{\dot{Q}_{\text{zto}}} = \frac{6861 + 4397}{22386} = 0,50
\]

d) **Process comparison**

<table>
<thead>
<tr>
<th>EXAMPLE</th>
<th>Outlet exhaust gas temperature</th>
<th>Heat regeneration</th>
<th>Efficiency of process</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>550</td>
<td>0</td>
<td>0,31</td>
</tr>
<tr>
<td>b)</td>
<td>426</td>
<td>3783</td>
<td>0,37</td>
</tr>
<tr>
<td>c)</td>
<td>180</td>
<td>10992</td>
<td>0,50</td>
</tr>
</tbody>
</table>
7. Combined (gas-steam) cycle

Analyse the combined cycle power plant in the figure and find the energy efficiencies of the following plant components:

- air compressor
- gas turbine
- steam superheater
- evaporator
- water heater (economizer)
- high pressure steam turbine
- low pressure steam turbine
- entire steam turbine
- feed water storage tank

For superheater and economizer also calculate the effectiveness of heat exchanger.
Known data

<table>
<thead>
<tr>
<th>Paramaters</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td></td>
</tr>
<tr>
<td>mass flow</td>
<td>( m_{\text{a}} = 55 ) kg/s</td>
</tr>
<tr>
<td>temperature at compressor inlet (ambient)</td>
<td>( T_{a1} = 10 ) °C</td>
</tr>
<tr>
<td>pressure at compressor inlet (ambient)</td>
<td>( p_{a1} = 1 ) bar</td>
</tr>
<tr>
<td>temperature at compressor outlet</td>
<td>( T_{a2} = 420 ) °C</td>
</tr>
<tr>
<td>pressure at compressor outlet</td>
<td>( p_{a2} = 15 ) bar</td>
</tr>
<tr>
<td>specific heat</td>
<td>( c_{p,a} = 1.005 ) kJ/kgK</td>
</tr>
<tr>
<td>isentropic exponent</td>
<td>( \kappa_a = 1.4 )</td>
</tr>
<tr>
<td>gas constant</td>
<td>( R_a = 287 ) J/kgK</td>
</tr>
<tr>
<td>Fuel</td>
<td></td>
</tr>
<tr>
<td>heating value</td>
<td>( H_s = 51 ) MJ/kg</td>
</tr>
<tr>
<td>exergy value</td>
<td>( e_f = 51 ) MJ/kg</td>
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<tr>
<td>mass flow</td>
<td>( m_{\text{f}} = 1.1 ) kg/s</td>
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<tr>
<td>Flue gas</td>
<td></td>
</tr>
<tr>
<td>specific heat</td>
<td>( c_{p,g} = 1.155 ) kJ/kgK</td>
</tr>
<tr>
<td>isentropic exponent</td>
<td>( \kappa_g = 1.4 )</td>
</tr>
<tr>
<td>gas constant</td>
<td>( R_g = 294 ) J/kgK</td>
</tr>
<tr>
<td>temperature at gas turbine outlet</td>
<td>( T_{g2} = 640 ) °C</td>
</tr>
<tr>
<td>temperature at superheater outlet</td>
<td>( T_{g3} = 540 ) °C</td>
</tr>
<tr>
<td>temperature at evaporator outlet</td>
<td>( T_{g4} = 315 ) °C</td>
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<tr>
<td>temperatur at water heater outlet</td>
<td>( T_{g5} = 200 ) °C</td>
</tr>
<tr>
<td>Water</td>
<td></td>
</tr>
<tr>
<td>feed water pressure</td>
<td>( p_{s1} = 80 ) bar</td>
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<tr>
<td>feed water temperature</td>
<td>( T_{s1} = 135 ) °C</td>
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<tr>
<td>superheater steam temperature</td>
<td>( T_{s4} = 500 ) °C</td>
</tr>
<tr>
<td>mass flow</td>
<td>( m_{\text{s}} = 9.9 ) kg/s</td>
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<tr>
<td>extraction pressure</td>
<td>( p_{s5} = 15 ) bar</td>
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<tr>
<td>extraction temperature</td>
<td>( T_{s5} = 290 ) °C</td>
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<tr>
<td>extracted steam mass flow</td>
<td>( m_{\text{ex}} = 1.45 ) kg/s</td>
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<tr>
<td>pressure at steam turbine outlet</td>
<td>( p_{s6} = 0.06 ) bar</td>
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<tr>
<td>steam dryness at turbine outlet</td>
<td>( x_{s6} = 0.89 )</td>
</tr>
<tr>
<td>steam dryness at condenser outlet</td>
<td>( x_{s7} = 0 )</td>
</tr>
<tr>
<td>pressure at condensate pump outlet</td>
<td>( p_{s8} = 3 ) bar</td>
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<tr>
<td>temperature at condensate pump outlet</td>
<td>( T_{s8} = 36 ) °C</td>
</tr>
<tr>
<td>steam dryness at feed water tank outlet</td>
<td>( x_{s9} = 0 )</td>
</tr>
</tbody>
</table>