Methodology for processing pressure traces used as inputs for combustion analyses in diesel engines

Davor Rašić*, Rok Vihar, Urban Žvar Baškovič, Tomaž Katrašnik

Faculty of Mechanical Engineering, University of Ljubljana, Aškerčeva 6, SI-1000 Ljubljana, Slovenia

*corresponding author

Abstract

This study proposes a novel methodology for designing an optimum equiripple finite impulse response (FIR) filter for processing in-cylinder pressure traces of a diesel internal combustion engine, which serve as inputs for high-precision combustion analyses. The proposed automated workflow is based on an innovative approach of determining the transition band frequencies and optimum filter order. The methodology is based on discrete Fourier transform analysis, which is the first step to estimate the location of the pass-band and stop-band frequencies. The second step uses short-time Fourier transform analysis to refine the estimated aforementioned frequencies. These pass-band and stop-band frequencies are further used to determine the most appropriate FIR filter order. The most widely used existing methods for estimating the FIR filter order are not effective in suppressing the oscillations in the rate-of-heat-release (ROHR) trace, thus hindering the accuracy of combustion analyses. To address this problem, an innovative method for determining the order of an FIR filter is proposed in this study. This method is based on the minimization of the integral of normalized signal-to-noise differences between the stop-band frequency and the Nyquist frequency. Developed filters were validated using spectral analysis and calculation of the ROHR. The validation results showed that the filters designed using the proposed innovative method were superior compared with those using the existing methods for all analyzed cases.

Highlights

- Pressure traces of a diesel engine were processed by first impulse response (FIR) filters with different orders
- Transition band frequencies were determined with an innovative method based on discrete Fourier transform and short-time Fourier transform
- Spectral analyses showed deficiencies of existing methods in determining the FIR filter order
- A new method of determining the FIR filter order for processing pressure traces was proposed
- The efficiency of the new method was demonstrated by spectral analyses and calculations of rate-of-heat-release traces

Keywords: Diesel engine, DFT, STFT, FIR filter, harmonics, spectral analyses

Abbreviations

CLCC – closed-loop combustion control
EGR – exhaust gas recirculation
1. Introduction

The number of modern diesel internal combustion engines being equipped with a built-in pressure sensor that provides an insight into in-cylinder combustion phenomena is increasing. The information on the in-cylinder pressure serves as an input to the thermodynamic analysis of in-cylinder processes, and can thus be considered as a prerequisite to an efficient closed-loop combustion control (CLCC). Currently, in-cylinder pressure measurements are mostly used only for monitoring the aging of components that can affect the operation of the engine and for adapting control strategies and injection methods according to changes of the fuel cetane number. Combustion control is being considered as one of the primary solutions to optimize the trade-off between improved efficiency and exhaust emission of an engine, in particular, during real driving conditions. In addition to these online tasks, in-cylinder pressure signals are also widely used in offline analyses during research and development activities. To efficiently support all required tasks, adequately preprocessing, i.e., filtering the pressure traces is of utmost importance. Filtering aims to suppress the unwanted contributions of vibrational eigenmodes in the combustion chamber as well as measurement noise while preserving the contributions of piston kinematics (i.e., pseudomotored contributions) and combustion.

The primary source of resonant pressure oscillations in the combustion chamber of a diesel engine is the rapid heat release during combustion of the premixed charge, which excites the gas in the combustion chamber with its acoustic resonance frequencies as analyzed in e.g., [1–4]. Secondary sources of resonant pressure oscillations are cycle-to-cycle variations of quantities of injected fuel and air [12], ignition delay, EGR rate [13], and flow dynamics in the cylinder.

Combustion analysis on the basis of the heat release law is susceptible to recorded resonant pressure oscillations sources, which are generally also amplified in ROHR analyses, where, besides the in-cylinder pressure, the in-cylinder pressure derivative is also an input to the thermodynamic model for evaluation of the ROHR. For illustration purposes, shown below is the basic form of the heat release equation [14]:

\[
\frac{dQ_r}{dt} = \left( \frac{\gamma}{\gamma - 1} \right) \frac{dp}{dt} + \frac{1}{\gamma - 1} \frac{V}{\gamma - 1} \frac{dV}{dt},
\]

where \( p \) represents the in-cylinder pressure, \( V \) is the current volume, \( \gamma \) is the specific heat ratio \( \frac{c_p}{c_v} \), and \( t \) is the time.

The presence of the pressure derivative in Eq. (1) indicates that any high-frequency noise or vibrational eigenmodes in the measured in-cylinder pressure will be manifested as an amplified distortion in the ROHR calculation. Fig. 1 shows the effect of pressure irregularities on the
calculated ROHR trace for an online type of analysis of the investigated light-duty engine (Table 1), where only one cycle is processed. It is evident that such an ROHR trace cannot be used in further calculations or for control purposes. Therefore, elimination or strong attenuation of noise and vibrational eigenmodes is required to allow the determination of different thermodynamic parameters and ROHR derivatives with higher fidelity.

Fig. 1. Measured unfiltered pressure trace and corresponding rate of heat release (ROHR) calculated at 2000 min⁻¹ and 100 N·m.

Elimination of noise and oscillations from pressure traces was addressed in several studies [15–19]. Payri et al. presented several papers on filtering of the in-cylinder pressure in diesel engines [15–17]. In [15], a method for analyzing the in-cylinder pressure was introduced, where the measured in-cylinder pressure was decomposed into three main signals: pseudomotored, combustion, and resonance. In [16] and [17], a method for determining cutoff frequencies for filtering in-cylinder pressures based on discrete Fourier transform (DFT) was introduced and the results were validated by means of standard deviation of pressure traces. In [16], a frequency spectrum and harmonic sequence-based method is proposed, which serves as a basis for calculating the difference between the value of the harmonic $k \cdot n_c$ and the average value of the harmonics between $(k \cdot n_c) + 1$ and $(k + 1) \cdot n_c - 1$, where $k$ represents the instantaneous harmonic and $n_c$ is the number of processed consecutive cycles. Such a calculation provides insightful information on the signal-to-noise ratio. In [16], a summary of different approaches for determining the required number of cycles to successfully suppress the irregularities introduced by cycle-to-cycle variations was presented. It can be concluded that there is no general consensus on this subject, as the number of cycles depends on the engine type, acquisition system, and operating point of the engine. Furthermore, an adaptive method for determining the optimal number of cycles based on the variation of the standard deviation of average cycles was also proposed; it was reported that for a direct-injection diesel engine, the
Cite paper as: RAŠIĆ, Davor, VIHAR, Rok, ŽVAR BAŠKOVIĆ, Urban, KATRAŠNIK, Tomaz. Methodology for processing pressure traces used as inputs for combustion analyses in diesel engines. *Measurement Science and Technology, Volume 28, Number 5 (2017)*, ISSN 0957-0233. doi: https://doi.org/10.1088/1361-6501/aa5f9e

optimal number of such cycles is 25 [16]. Based on analysis of existing publications, it can be concluded that existing techniques for noise and vibrational eigenmodes removal were not applied in online analyses while processing only a single-pressure trace. Furthermore, they were not validated with ROHR traces, being one of the primary parameters affected by the filtering of in-cylinder pressure traces.

Various filtering techniques can be used for pressure signal processing. Because of their ability to retain information and their simultaneous high-frequency noise removal, low-pass filters are frequently used. The most commonly used filter is the equiripple FIR type filter [18, 21], where the efficiency is driven by selection of an appropriate order and transition band frequencies (this term is further used to denote the pass-band and stop-band frequencies). The main advantage of the equiripple filter is the elimination of the Gibbs effect, which can drastically distort the filtered pressure trace [24]. A key factor in the successful application of a low-pass filter is the determination of the pass-band and stop-band frequencies above which it is assumed that only noise is present [20, 21].

There are several methods for an approximate determination of the FIR filter order, e.g., the Parks-McClellan algorithm, Kaiser equation, Hermann equation, and Bellanger equation [11, 20–26]. These methods are relatively easy to use. Various authors described the Parks-McClellan algorithm as the most efficient and optimal method for determining the FIR filter order [20, 22–24]. Despite this fact, it is shown in this paper that all existing methods determine similar orders of the FIR filter. Furthermore, as shown in the presented analysis, these methods for determining orders of the filters do not successfully suppress the unwanted contributions of vibrational eigenmodes in the combustion chamber while preserving contributions of piston kinematics and combustion, which is required for high-fidelity ROHR calculations.

To address this problem, this paper presents a novel methodology for determining an optimum FIR filter (summarized in Appendix B) that comprises an innovative procedure for determining transition band frequencies and an optimum FIR filter order to be used for ROHR analyses of in-cylinder pressures. The innovative methodology for determining the transition band frequencies is based on a two-step procedure. In the first step, transition band frequencies are estimated using discrete Fourier transform (DFT), and afterward, they are refined in the second step using short-time Fourier transform (STFT) analysis. These frequency data are further used to determine an optimum order of equiripple FIR filters by an innovative method that is based on the minimization of the integral of normalized signal-to-noise differences between the stop-band frequency and the Nyquist frequency at a given operating point. Such a procedure can be implemented in an automated way using a small number of input parameters that are maintained constant in all analyzed cases in the present study.

The efficiency of the innovative method is proven by the higher quality of ROHR traces calculated using the proposed method, compared with the ROHR traces calculated using the aforementioned existing methods for determining FIR filter orders. The comparison is performed for single cycles, i.e., the most extreme online application case of the CLCC, and for average cycles serving as examples of offline analyses. The universal applicability of the innovative method is demonstrated on two significantly different engines, i.e., a light and a heavy-duty diesel engine.

The paper is organized as follows: Section 2 provides basic information on the experimental setup; Section 3 first introduces the methods for processing in-cylinder pressure traces and then presents the innovative methodology for determining transition band frequencies; Section 4 first shows an analysis of existing methods for determining orders of the filters and then the analysis of the proposed innovative method for determining the FIR filter order that relies on the transition band frequencies determined as described in Section 3; and Section 5 presents the validation of the proposed methodology, which is summarized in Appendix B on a wide variety of operating conditions.
2. Experimental setup

The experimental work was performed on two engines, namely a 4-cylinder, 4-stroke, turbocharged, 1.6 L EURO4 light-duty diesel engine (hereinafter referred to as Engine 1), and a 6-cylinder, 4-stroke, turbocharged, 6.87 L heavy-duty MAN diesel engine model D 0826 LOH 15 (hereinafter referred to as Engine 2). The main characteristics of Engine 1 and Engine 2 are presented in Appendix A. In-cylinder pressure measurements were performed on both engines at various operating points. For brevity, Table 1 highlights only the operating points directly used to present the methodology for processing the measured pressure traces. The resolution of pressure acquisition was 0.1°CA and 1°CA for Engine 1 and 0.2°CA and 1°CA for Engine 2. Both engines were coupled to a Zöllner B-350AC eddy-current dynamometer controlled by Kristel, Seibt & Co control system KS ADAC. A Kistler CAM UNIT Type 2613B shaft encoder provided an external trigger and an external clock (0.1–1°CA) for the data acquisition system. In-cylinder pressures were measured with calibrated piezoelectric pressure transducer AVL GH14D in combination with charge amplifier AVL MICROIFEM, connected to a 16-bit, 4-channel National Instruments data acquisition system with a maximum sampling frequency of 1 MS/s per channel. The top dead center was determined using a capacitive sensor COM Type 2653. The general scheme of the experimental setup is shown in Fig. 2.

![Schematic diagram of the experimental setup.](image)

<table>
<thead>
<tr>
<th>Operating point</th>
<th>Engine speed [^{\text{min}^{-1}}]</th>
<th>Engine load [N \cdot m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1.1</td>
<td>1200</td>
<td>20</td>
</tr>
<tr>
<td>E1.2</td>
<td>1200</td>
<td>100</td>
</tr>
<tr>
<td>E1.3</td>
<td>2000</td>
<td>20</td>
</tr>
<tr>
<td>E1.4</td>
<td>2000</td>
<td>100</td>
</tr>
<tr>
<td>E1.5</td>
<td>3000</td>
<td>20</td>
</tr>
</tbody>
</table>
E1.6 3000 100
E1.7 3000 160
E2.1 1500 280
E2.2 2400 370

3. Methodology for determination of transition band frequencies

In this section, the methods for processing in-cylinder pressure traces and examples of data analyses are introduced first. This forms the basis for the derivation of an innovative generic methodology for determining transition band frequencies. For brevity, the methodology is demonstrated on one operating point of Engine 1.

3.1. Methods for processing in-cylinder pressure traces

For analysis purposes, cycles measured in an operating point were concatenated in a continuous periodic pressure trace. In the presented analysis, 25 cycles were collected in every operating point; this by no means restricts the applicability of the method to any other meaningful number of cycles as discussed in the introduction. This procedure is indicated as transition band frequency (hereinafter referred to as TBF) Step 1 in the flow chart for determining an optimum FIR filter order presented in Appendix B.

The methods of determining transition band frequencies rely on a combination of two methodologies for processing in-cylinder pressure traces:

I. DFT analysis (TBF Step 2.1 in Appendix B) of pressure traces aimed at determining signal-to-noise ratio as proposed by Payri et al. [16]

DFT divides the input signal of length N in two output signals, which contain the amplitude of sine and cosine signals. The input signal is in the time domain in general, while the amplitude of sine and cosine signals is calculated in the frequency domain. The variable N is a positive integer and is typically a power of 2 (e.g., 128, 256, 512, and 1024) and represents the number of points in the time domain. The first reason for such a feature is that digital data processing operates in binary code, while the second reason is that the most efficient way of calculating the DFT is using the algorithm of fast Fourier transform (FFT), which normally operates with N being a power of 2. The abscissa of the signal in the frequency domain can be commonly represented in four different ways [20], whereas for the purpose of this study the abscissa is analogous to frequency.

II. STFT analysis (TBF Step 2.2 in Appendix B) of pressure traces

By analyzing the signal using Fourier transform (FT), the interrelation to the temporal scale is lost. In-cylinder processes are characterized by large variability of thermodynamic parameters, and thus, have also different contributions to the pressure trace. It is therefore important to also obtain information on the temporal appearance of particular frequencies. To achieve this, pressure traces are additionally processed using STFT [20,21,25], which divides the signal in the time domain to a 2D time-frequency domain, thus providing insights into frequency variations within each window. This is achieved by the application of a time-localized FT on a window of a certain length that
transverses over the pressure trace, following a temporal evolution of the signal. An important factor of STFT analysis is the length selection of the analyzed windows. On one hand, longer windows contain more information; however, this information is averaged over longer periods causing a potential loss of transient and nonstationary phenomena. On the other hand, shorter windows contain less information, which causes lower frequency resolutions, but allows for a more accurate time localization of frequency spectra. For the purpose of the following STFT analysis, the Hanning window [20–25] function was used.

3.2. Data analysis

To provide an insight on the frequency spectra at different operating regimes, the results of DFT of 25 concatenated cycles are analyzed for various engine speeds at constant engine load. For illustrative purposes, the analysis in this section will be performed only on the data for Engine 1 operating points acquired with the 0.1°CA resolution. The corresponding sampling frequency for data at 3000 min⁻¹ is 180 kHz, at 2000 min⁻¹ is 120 kHz, and at 1200 min⁻¹ is 72 kHz.

Fig. 3a shows the DFT analysis of 25 concatenated cycles (n_c) at constant engine load and varying engine speed. It is evident from the figure that harmonics, being integer multiplications of n_c, feature distinctly higher values at low harmonic numbers as they correspond to characteristics of a particular cycle of which n_c values have been analyzed. On the other hand, harmonics that are not integer multiplications of n_c, with amplitudes that are likely to be zero in an ideal case, correspond to irregularities as, for example, noise caused by thermal effects, sensor resonance, and its nonlinearity, vibrations, and signal drift. These features characterize such DFT analysis of multiple cycles as very suitable for indicating the distinction between the signal and noise as proposed in [16]. All figures thus clearly show the contribution of piston kinematics at low harmonics being integer multiplications of n_c, followed by contributions of combustion and subsequently by contributions of vibrational eigenmodes that are specifically indicated in all figures. It can also be observed that at higher frequencies, the amplitude of harmonics that are not integer multiplications of n_c feature the same amplitude as those which are integer multiplications of n_c, indicating the noise contribution of these harmonics. When analyzing the amplitude of vibrational eigenmodes at different engine speeds, it can be concluded that their relative amplitude increases with the increase of engine speed above approximately the 500th harmonic. Furthermore, it can be observed that the 500th harmonic is the lowest frequency range of combustion contribution to the frequency spectra. This is the consequence of longer ignition delay period in terms of crankshaft revolution resulting in larger amount of the fuel being injected in the ignition delay period leading to a more intense heat release of the premixed charge, and thus to more intense excitation of vibrational eigenmodes.
Fig. 3. DFT analysis of a sequence of 25 cycles: a.) at constant engine load and varying engine speed (E1.2, E1.4, E1.6), b.) at constant engine speed and varying engine load (E1.3, E1.4, E1.5)

Fig. 3b shows the DFT analysis of \( n_c \) concatenated cycles with variation of engine load at constant engine speed. It can be observed that the amplitude of vibrational eigenmodes increases with increasing load above approximately the 400\(^{th}\) harmonic. Furthermore, it can be observed that the 500\(^{th}\) harmonic is the lowest frequency range of combustion contributions to the frequency spectra. Increases in amplitudes of vibrational eigenmodes are associated with more energy supplied by the fuel, and thus higher potential to excite the eigenmodes. In addition, it can be noted that increasing the engine load results in increasing amplitudes of vibrational eigenmodes at higher frequencies, which is mainly related to higher in-cylinder temperatures.

Based on Fig. 3, it can be concluded that both engine speed and load influence the amplitude and frequencies of vibrational eigenmodes. By analyzing the DFT of \( n_c \) consecutive cycles on the basis of harmonics in the spectrum, additional information on the signal-to-noise ratio can be obtained. A calculation of the difference between the amplitude of the instantaneous value of a harmonic \( k \cdot n_c \) and the average value of harmonics between \((k \cdot n_c) + 1\) and \((k + 1) \cdot n_c - 1\) can provide the relationship between signal-to-noise ratio being presented in Fig. 4 [16]:

\[
y = DFT_A(k \cdot n_c) - \frac{\sum_{i=(k \cdot n_c) + 1}^{(k + 1) \cdot n_c - 1} DFT_A(i)}{((k + 1) \cdot n_c - 1) - ((k \cdot n_c) + 1)}
\]  

(2)

where \( DFT_A(k \cdot n_c) \) represents the DFT amplitude of the instantaneous harmonic \( k \cdot n_c \), \( DFT_A(i) \) the DFT amplitude summation of harmonics between \((k \cdot n_c) + 1\) and \((k + 1) \cdot n_c - 1\), \( k \) the discussed harmonic, and \( n_c \) the number of processed concatenated cycles.

To provide a more general representation, the signal-to-noise ratio amplitude was normalized to a maximum value of 100 denoted as \( A_{max} \) (TBF Step 3 in Appendix B). The conversion between the harmonic and the frequency domain is given by [16]
where $k$ represents the discussed harmonic, $n_c$ the number of processed concatenated cycles, $n_{pc}$ the number of samples per cycle, and $f_s$ the sampling frequency.

In Fig. 4, it is evident that when the signal-to-noise ratio approaches small values, it can be assumed that the signal mostly consists of noise and vibrational eigenmodes.

**3.3. Criteria for determination of transition band frequencies**

This section introduces an innovative methodology for determining the transition band frequencies. The methodology is designed in a way that it can be implemented in a computer code, and thus can be used for an automatic determination of transition band frequencies.

DFT is commonly used in studies on attenuation of noise and resonant pressure oscillations [15–17] as it provides a basic insight on the signal-to-noise ratio. Therefore, in the proposed innovative methodology, DFT (TBF Step 4.1 in Appendix B) is used first to estimate the transition band frequencies. However, for an accurate evaluation of ROHR, transition band frequencies relevant to the combustion process must be accurately determined. Therefore, based on the estimated values of transition band frequencies obtained from TBF Step 4.1, the final transition band frequencies of the combustion process are determined using a refinement procedure based on STFT (TBF Step 4.2 in Appendix B). The combination of both analyses provides a comprehensive tool for analyzing in–cylinder processes with the capability to identify variations within each cycle and between cycles. This procedure thus provides the basis for effective suppression of unwanted contributions of vibrational eigenmodes in the combustion chamber and measurement noise, while preserving the contribution of piston kinematics and combustion.
The transition band frequencies estimated using the DFT analysis are defined as the intervals between the first minimum of the signal-to-noise ratio and the first major vibrational eigenmode. The mathematical formulation of these two criteria is given as follows:

- The pass-band frequency estimation \(PBF_e\) depicted by the first red line in Fig. 4 is defined by the first local minimum (\(y'_{first} = 0\)), at which the normalized difference \(y\) reaches a value below the prespecified threshold \((y < A \cdot A_{max})\):  
  \[PBF_e = \{v \mid y'(v)_{first} = 0 \land y(v) < A \cdot A_{max}\},\]  
  \(\text{(4)}\)

where the value of \(A\) in the presented analysis is set as 0.001 for all operating points and engines, and \(A_{max}\) is 100 as given is Section 3.2.

The main objective of the \(PBF_e\) is to locate the minimum frequency of a vibrational eigenmode, which is used as input to the refinement step of determining the pass-band frequency using STFT.

- The stop-band frequency estimation \(SBF_e\) depicted by the second red line in Fig. 4 is defined as the last minimum (\(y'(v)_{last} = 0\)), after which the value of \(y\) exceeds the prespecified threshold \((y(v > SBF_e) > B \cdot A_{max})\):  
  \[SBF_e = \{v \mid y'(v)_{last} = 0 \land y(v > SBF_e) > B \cdot A_{max}\},\]  
  \(\text{(5)}\)

where the value of \(B\) in the presented analysis is set as 0.0025 for all operating points and engines.

The main objective of the \(SBF_e\) is to locate the frequency where the first major vibrational eigenmode arises, which is used as input to the refinement step of determining the stop-band frequency using STFT.

DFT analysis provides a straightforward estimation of transition band frequencies for pressure signals at various operating points; however, it cannot be stated with certainty that the estimated transition band frequencies are relevant to the combustion period as the DFT analysis does not provide an insight on the frequency spectra in the time domain. Therefore, a refinement step is performed using the STFT analysis. The primary purpose of applying STFT is to refine the proposed transition band frequencies derived from the DFT analysis (TBF Step 4.2 in Appendix B). Refinement of transition band frequencies using STFT is based on the following criteria:

- Refinement step of the pass-band frequency \(PBF_r\) checks if, at \(PBF_e\), any of the analyzed cycles features power/frequency (denoted as \(PF\)) intensity \([5]\) larger than a prespecified maximum value \(C\) in the combustion period-related part of the engine cycle and decreases \(PBF_r\) value in a way that \(PF_i(v) < C\):  
  \[PBF_r = \{\min(v) \mid PF_i(v) < C, i = \{1, ..., n_c\}\},\]  
  \(\text{(6)}\)

where the value of \(C\) is set as \(-55\) dB/Hz in the presented analysis.

- Refinement step of the stop-band frequency \(SBF_r\) checks if, starting with \(SBF_e\) within all analyzed cycles, the absolute difference between the absolute value of \(PF\) of the cycle with the maximum \(PF\) \((|\max(PF_i(v))|)\) and the absolute value of \(PF\)
of the cycle with the minimum $PF$ ($|\min(PF_i(v))|$) exceed the prespecified threshold $D$ and, if necessary, decreases the $SBF$ in a way to meet this criterion.

\[ SBF = \left\{ \min(v), \left| \max(PF_i(v)) \right| - \left| \min(PF_i(v)) \right| > D, i \in \{1, \ldots, n_c\} \right\}, \]  

where the value of $D$ is set as 10 dB/Hz in the presented analysis.

### 3.4. Determination of transition band frequencies

The estimation and refinement steps for the determination of transition band frequencies are shown in Fig. 5, where a direct comparison between DFT and STFT analysis is presented. The red lines represent the transition band frequencies estimated by DFT using the criteria outlined in Section 3.3, where low frequency corresponds to the pass-band frequency and high-frequency to the stop-band frequency. Similarly, the green lines represent the transition band frequencies refined by STFT using the criteria outlined in Section 3.3. For illustrative purposes, several cycles on the STFT spectrogram were analyzed with data cursors.

Fig. 5. STFT (left) and DFT (right) analysis for operating point E1.4. STFT – Data cursors represent $PBF$ and $SBF$ related data at respective frequencies; DFT – Red dotted line represents $PBF$, whereas the red line represents $SBF$. The green line represents $SBF$, whereas the green dotted line represents the refined value of the stop-band frequency $PBF$.

It is evident from the STFT analysis in Fig. 5 that all cycles are not identical, and that differences are most pronounced in the frequency range corresponding to vibrational eigenmodes. Fig. 5 thus illustrates the two most distinct cycles in terms of intensity of excited vibrational eigenmodes at the sensor location (denoted by the cycle with low or high eigenfrequency intensity) being pinpointed by the STFT analysis. The main reason for the differences between the cycles arises from the random angular position of the fuel ignition belonging to one of the injected sprays. Therefore, vibrational eigenmodes that feature angular dependency are not excited repeatedly in terms of the angular orientation, and thus of the sensor position, which results in cycle-to-cycle variations of the measured vibrational eigenmodes. It can be observed further in Fig. 5 that the value of $PBF$ is 1610 Hz. Starting from this value, it is apparent that the minimum frequency where the $PF$ of all analyzed 25 cycles decreases
below the prespecified threshold $C$ ($-55$ dB/Hz) is equal to $1610$ Hz. Specifically, the data cursor at $1.302$ s and $PF$ of $-55$ dB/Hz represents the minimum frequency where the $PF$ of all cycles decreases below $-55$ dB/Hz. It can thus be concluded that refinement of the pass-band frequency was not required in the analyzed case.

The value of $SBF_e$ is $5410$ Hz. Starting from this value, it can be observed that the minimum frequency where the $PF$ of all cycles decreases below $-55$ dB/Hz is equal to $1610$ Hz. Specifically, the data cursor at $1.302$ s and $PF$ of $-55$ dB/Hz represent the minimum frequency where the $PF$ of all cycles decreases below $-55$ dB/Hz. It can thus be concluded that refinement of the pass-band frequency was not required in the analyzed case.

The refined transition band frequency interval for this operating point is thus $1610–4130$ Hz. This analysis demonstrates the benefits of consecutively applying the DFT and STFT analyses to determine and refine the transition band frequency interval. Additionally, the STFT analysis also shows nonhomogeneous cycle-to-cycle variations in the $PF$ intensity of the eigenfrequencies, which can clearly be observed in the frequency interval between $4000$ Hz and $7000$ Hz.

Using this methodology, transition band frequencies were determined for both the analyzed engines and corresponding operating points: Engine 1 - $1200 \text{ min}^{-1}$ and $20 \text{ N} \cdot \text{m}$ (E1.1); $3000 \text{ min}^{-1}$ and $160 \text{ N} \cdot \text{m}$ (E1.7); Engine 2 - $1500 \text{ min}^{-1}$ and $280 \text{ N} \cdot \text{m}$ (E2.1); $2400 \text{ min}^{-1}$ and $370 \text{ N} \cdot \text{m}$ (E2.2). To demonstrate the wide applicability of the methodology, two sampling frequencies are analyzed per operating point. The input data of FIR filter order calculations for all analyzed cases are summarized in Table 2 for Engine 1 and in Table 3 for Engine 2.

### Table 2. Engine 1 input data for FIR filter order calculations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0.1°CA sampling frequency</th>
<th>1°CA sampling frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E1.1</td>
<td>E1.7</td>
</tr>
<tr>
<td>Sampling frequency [kHz]</td>
<td>72</td>
<td>180</td>
</tr>
<tr>
<td>Pass-band frequency [Hz]</td>
<td>700</td>
<td>1640</td>
</tr>
<tr>
<td>Stop-band frequency [Hz]</td>
<td>3650</td>
<td>3750</td>
</tr>
<tr>
<td>Pass-band ripple [/]</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Stop-band ripple [/]</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

### Table 3. Engine 2 input data for FIR filter order calculations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0.2°CA sampling frequency</th>
<th>1°CA sampling frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E2.1</td>
<td>E2.2</td>
</tr>
<tr>
<td>Sampling frequency [kHz]</td>
<td>45</td>
<td>72</td>
</tr>
<tr>
<td>Pass-band frequency [Hz]</td>
<td>2000</td>
<td>2200</td>
</tr>
<tr>
<td>Stop-band frequency [Hz]</td>
<td>3900</td>
<td>4000</td>
</tr>
<tr>
<td>Pass-band ripple [/]</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Stop-band ripple [/]</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>
4. Evaluation of FIR filter order

In this section, an analysis of existing methods for determining orders of the FIR filters is presented first, followed by an elaboration of the innovative method for determining the FIR filter order. For consistency, both analyses rely on the transition band frequencies determined in Section 3.

4.1. Existing methods for determining FIR filter order

As mentioned in the introduction, the most commonly used method for determining equiripple FIR filter order is the Parks-McClellan algorithm (Eq. (C. 4) in Appendix C); in addition, the filter order is also determined using the Kaiser method (Eq. (C. 1)), Hermann method (Eq. (C. 2)), and Bellanger method (Eq. (C. 3)). The input data for all methods were extracted from Tables 2 and 3.

Table 4. FIR filter order evaluated with different methods for selected Engine 1 operating points with different sampling frequencies

<table>
<thead>
<tr>
<th>Operating point</th>
<th>0.1°CA sampling frequency</th>
<th>1°CA sampling frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>E1.1</td>
<td>E1.7</td>
</tr>
<tr>
<td>Parks-McClellan</td>
<td>46</td>
<td>169</td>
</tr>
<tr>
<td>Kaiser</td>
<td>44</td>
<td>160</td>
</tr>
<tr>
<td>Hermann</td>
<td>45</td>
<td>168</td>
</tr>
<tr>
<td>Bellanger</td>
<td>47</td>
<td>173</td>
</tr>
</tbody>
</table>

Table 5. FIR filter order evaluated with different methods for selected Engine 2 operating points with different sampling frequencies

<table>
<thead>
<tr>
<th>Operating point</th>
<th>0.2°CA sampling frequency</th>
<th>1°CA sampling frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>E2.1</td>
<td>E2.2</td>
</tr>
<tr>
<td>Parks-McClellan</td>
<td>46</td>
<td>79</td>
</tr>
<tr>
<td>Kaiser</td>
<td>46</td>
<td>75</td>
</tr>
<tr>
<td>Hermann</td>
<td>46</td>
<td>77</td>
</tr>
<tr>
<td>Bellanger</td>
<td>47</td>
<td>80</td>
</tr>
</tbody>
</table>

The results in Table 4 and Table 5 indicate that the difference between the various approaches in FIR filter order calculations are marginal. The Parks-McClellan algorithm is described by several authors as the most efficient and optimal method for determining the FIR filter order [20, 22–24]. It is based on modification of the Remez exchange algorithm, which is a generic iterative procedure to optimally approximate any function, whereas the other approaches of determining the FIR filter order rely on approximative equations. The goal of the algorithm is to minimize the error in the pass-band and stop-band by utilizing the Chebyshev approximation. Considering the fact that the Parks-McClellan algorithm is favored over other existing methods
for determining the FIR filter order, it will subsequently be used to design the FIR filter and to represent the existing methods of determining the FIR filter order in further analyses.

Fig. 6. Spectral analysis comparison of unfiltered and filtered pressure traces for operating point E1.1 at 0.1°C. The red vertical line represents the stop-band harmonic.

Fig. 7. Spectral analysis comparison of unfiltered and filtered pressure traces for operating point E2.1 at 0.2°C. The red vertical line represents the stop-band harmonic.
Fig. 8. Spectral analysis comparison of unfiltered and filtered pressure traces for operating point E1.1 at 1°CA. The red vertical line represents the stop-band harmonic.

Fig. 9. Spectral analysis comparison of unfiltered and filtered pressure traces for operating point E2.1 at 1°CA. The red vertical line represents the stop-band harmonic.

Figs. 6 and 8 show the spectral analyses of a) unfiltered and b) filtered pressure traces for E1.1 while applying the filter designed based on the transition band frequencies presented in Table 2 and FIR filter order determined by the Parks-McClellan algorithm extracted from Table 4. Figs. 7 and 9 show the spectral analysis of a) unfiltered and b) filtered pressure traces for E2.1 while applying the filter designed based on the frequencies presented in Table 3 and FIR filter order determined by the Parks-McClellan algorithm extracted from Table 5. The results indicate that the FIR filters designed based on the Parks-McClellan algorithm method of determining FIR filter orders do not sufficiently filter out the undesirable higher frequencies related to vibrational eigenmodes and noise. Spectral analyses at operating points E1.7 and E2.2 show similar results. From [20–25], it can be concluded that the primary parameter that governs the performance of FIR filters is its order; hence it can be deduced that higher filter orders should be used to achieve satisfactory filtering.
4.2. Innovative method for FIR filter order determination

Based on the findings in the previous section, an innovative method for determining FIR filter order is proposed in this section. It comprises FIR filter order (hereinafter referred to as FFO) Step 1 to FFO Step 5 as described in Appendix B.

As the primary parameter that governs the performance of FIR filters is its order, satisfactory results could be achieved by determining the filter order more accurately. In addition, the normalized differences as shown in Fig. 4 provide reliable information on the signal-to-noise ratio.

Hence, the basis for an innovative filter originates from these two features, and thus it can be hypothesized that an efficient filter can be designed by minimizing the integral of the normalized signal-to-noise differences ($P$) between the stop-band frequency and the Nyquist frequency:

$$P = \int_{\omega_s}^{Nyq} D_n df,$$

where $D_n$ represents a series of discrete values of normalized difference, $\omega_s$ the stop-band frequency, and $Nyq$ the Nyquist frequency. The optimal filter order defined by the proposed approach is thus defined as the filter order that corresponds to a global minimum of the function $P$.

It was already pointed out in previous sections that the performance of FIR filters is determined by its order. An extensive analysis of the dependency of the function $P$ on the sampling frequency and transition band interval size is provided in Appendix D. The figures in Appendix D clearly indicate that the minimum value of the function $P$ and its position appear at different FIR filter orders for different sampling frequencies, transition band frequencies, and operating points. Owing to this large variety of independent parameters, we do not aim to propose an explicit formula for determining the optimum FIR filter order, but to propose a reliable automated workflow for finding the minimum value of the function $P$, i.e., the optimum FIR filter order for a particular engine operating point, which is characterized by a particular sampling frequency and transition band frequencies.

The main conclusion of the analysis presented in Appendix D is that for all analyzed cases, the value of the function $P$ first decreases with increasing FIR filter order until it reaches a minimum, after which it first gradually increases until it reaches the area of filter instability. It is also shown in Appendix D that qualitatively, the shape of the function $P$ is not dependent on sampling frequency, transition band interval size, or operating point. This characteristic is of utmost importance for our innovative method, which relies on the fact that the global minimum of the function $P$ lies in the continuous area of the function with ample margin toward the area of filter instability.

In between the decreasing function $P$ values and the minimum value, the values of the function might feature some nondistinct local minimum values, which emerge because of the relatively low FIR filter order step size used in the presented analysis. Using sufficiently large FIR filter order step sizes prevents the nondistinct local minimum values from emerging.

The workflow for determining an optimum FIR filter order using the proposed innovative method relies on the estimation steps (FFO Steps 1–5), which determine the interval containing the global minimum of the value of function $P$ and on the refinement steps (FFO Steps 5a–5d), which further determine the exact location of the global minimum of function $P$, and thus the optimum FIR filter order.

The first estimation step of the workflow is to design 1 low-pass FIR filters, $F_i$, with filter orders $N_i$ (FFO Step 1 in Appendix B), while considering the transition band frequencies given by...
The primary purpose of the estimation step is to locate the interval containing the global minimum by scanning the values of \( P(N_f) \). Therefore, the main requirement for the values \( N_1 \) and \( N_s \) is to ensure that a stable interval of filter orders is scanned with a sufficiently small number of \( N_i \) to ensure computational efficiency of the method. These requirements are also aligned with the requirement of sufficiently larger \( N_s \) values to prevent the solution from stopping at a potential local minimum. On the other hand, the values of \( N_s \) should be small enough to ensure at least a few \( N_i \) values in the stable interval of filter orders and to prevent too many refinement steps. Because of these facts and to ensure a high level of generality and robustness of the filter, Eq. (9) and also Eqns. (10) and (11) feature very simple ansatz functions as these are used only as supporting functions in the estimation step.

Transition band interval size dependent factor \( p_1 \) is defined by the following relation:

\[
p_1 = a_1 * y + a_2, \quad (10)
\]

where \( y \) represents the transition band interval size in Hz; the value of \( a_1 \) is set as \(-\frac{1.5 \times 10^{-7}}{Hz^2}\) in the presented analysis for all operating points and engines, and the value of \( a_2 \) is set as \(-\frac{8 \times 10^{-4}}{Hz}\) for all operating points and engines.

Transition band interval size dependent factor \( p_2 \) is defined by the following relation:

\[
p_2 = b_1 * y + b_2, \quad (11)
\]

where \( y \) represents the transition band interval size in Hz; the value of \( b_1 \) is set as \(-\frac{7 \times 10^{-4}}{Hz}\) in the presented analysis for all operating points and engines, and the value of \( b_2 \) is set as 7 for all operating points and engines.

The final FIR filter order \( N_F \) is determined by the FIR filter order \( N_{k+2} \). FIR filter order \( N_k \) is defined by the first positive derivative value of the function \( P \left( P'(N_s)_{\text{first}} > 0 \right) \). As previously mentioned, the values of the function \( P \) gradually increases before reaching the filter instability area after reaching the global minimum; therefore, the values of \( P \) for the following two filter order step size FIR filters \( N_{k+1} \) and \( N_{k+2} \) must be larger than their predecessors:

\[
N_F = \{N_{k+2} \mid N_k = P'(N_s)_{\text{first}} > 0 \land P(N_{k+2}) > P(N_{k+1}) > P(N_k) \}. \quad (12)
\]

The procedure that uses Eq. (12) in combination with adequate \( N_s \) values (that prevent solution stops at a potential local minimum) ensures that \( N_{k+1} \) and \( N_{k+2} \) are larger than the optimal filter order. This procedure is also sufficiently robust to ensure credible results also in the case if \( N_{k+1} \) and/or \( N_{k+2} \) pass the stable interval of filter orders, because, as presented in Appendix D, the values of \( P \) in the unstable area are larger than the minimum \( P \) value.
The primary purpose of the estimation step is to distinguish the area of the global minima of the function $P$, followed by the refinement step to identify the global minima of the function $P$ and the associated optimum FIR filter order. The optimum FIR filter order can correspond to one of the filters designed using the estimation steps of this workflow or to a filter with an intermediate order (FFO Step 5 in Appendix B). In order to determine the optimum FIR filter order, refinement steps are required:

- **FFO Step 5a:** The first refinement substep in locating the optimum filter is to locate the filter order $N_i$ of designed filters, which generates the minimal integral of the normalized signal-to-noise difference $P_i$. This value will serve as an initial boundary interval value for further steps:

  $$N_{i_1} = \text{MIN}(P_i(N_i)).$$  

- **FFO Step 5b:** The second refinement substep is to determine the second boundary interval filter order $N_{i_2}$ determined on the basis of the minimum $P_i(N_i)$ value in FFO Step 5a. $N_{i_2}$ is denoted as the minimum of $P_{i-1}(N_{i-1})$ and $P_{i+1}(N_{i+1})$:

  $$N_{i_2} = \text{MIN}(P_{i-1}(N_{i-1}), P_{i+1}(N_{i+1})).$$  

- **FFO Step 5c:** The third refinement substep is to determine an intermediate filter order using the bisection method:

  $$N_i = \frac{N_{i_1} + N_{i_2}}{2}.$$  

- **FFO Step 5d:** The last refinement substep is to design an FIR filter $F_i$ with filter order $N_i$:

  $$F_i(N_i).$$

If the designed FIR filter from FFO Step 5d does not feature a near zero or a satisfactory value of the function $P$, the refinement procedure of FFO Step 5 is repeated.

The data in Tables 6 and 7 indicate that the filter orders evaluated using the proposed method (values indicated by bold font) are higher than the filter orders evaluated using the Parks-McClellan algorithm (values indicated in italic font), whereas it can also be observed that the trends of both methods are similar. Very high filter orders certainly require more computational effort; however, considering the fact that 0.1°CA and 0.2°CA sampling frequencies mainly correspond to the offline analyses and that sampling frequency in the range of 1°CA can be considered as representative for online analyses, it can be concluded that the proposed filter orders generally do not turn out to be too demanding for the hardware being used in the particular application. FIR filters evaluated using the proposed method feature a significantly lower value of the function $P$ (Eq. (8)) in comparison with the filters evaluated using the Parks-McClellan algorithm, which is also evident from Figs. 11 and 12. These statements are further confirmed...
Table 6. FIR filter orders and corresponding integral value $P$ for Engine 1 defined operating points (italic font – Parks-McClellan algorithm, bold font – new proposed method)

<table>
<thead>
<tr>
<th>Filter order</th>
<th>$P$</th>
<th>Filter order</th>
<th>$P$</th>
<th>Filter order</th>
<th>$P$</th>
<th>Filter order</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>46</td>
<td>54.67</td>
<td>169</td>
<td>115.19</td>
<td>16</td>
<td>22.06</td>
<td>22</td>
<td>75.15</td>
</tr>
<tr>
<td>200</td>
<td>0.053</td>
<td>613</td>
<td>0.107</td>
<td>52</td>
<td>0.091</td>
<td>94</td>
<td>0.089</td>
</tr>
</tbody>
</table>

Table 7. FIR filter orders and corresponding integral value $P$ for Engine 2 defined operating points (italic font – Parks-McClellan algorithm, bold font – new proposed method)

<table>
<thead>
<tr>
<th>Filter order</th>
<th>$P$</th>
<th>Filter order</th>
<th>$P$</th>
<th>Filter order</th>
<th>$P$</th>
<th>Filter order</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>46</td>
<td>96.51</td>
<td>79</td>
<td>35.01</td>
<td>18</td>
<td>7.861</td>
<td>32</td>
<td>6.240</td>
</tr>
<tr>
<td>200</td>
<td>0.021</td>
<td>269</td>
<td>0.063</td>
<td>62</td>
<td>0.015</td>
<td>92</td>
<td>0.088</td>
</tr>
</tbody>
</table>
Fig. 10. Spectral analysis of filtered pressure traces based on the proposed innovative method for determining the FIR filter order at various operating points: a.) E1.1 at 0.1°CA sampling frequency, b.) E2.1 at 0.2°CA sampling frequency, c.) E1.1 at 1°CA sampling frequency, d.) E2.1 at 1°CA sampling frequency. The red vertical line represents the stop-band harmonic.

Fig. 10 shows the spectral analysis of filtered pressure traces for E1.1 and E2.1 at different sampling frequencies (°CA) while applying the filter designed based on the frequencies presented in Table 2 and filter order determined by the proposed innovative method. Comparison of Fig. 10 and Figs. 6b, 7b, 8b, and 9b indicates that the FIR filters designed based on the proposed method outperform that of Parks-McClellan algorithm for evaluating FIR filter order and effectively filter out undesirable higher frequencies related to vibrational eigenmodes and noise of the entire pool of selected cycles at various sampling frequencies; this confirms the general applicability of the proposed method. Spectral analyses at operating points E1.7 and E2.2 show similar results.
4.3. Comparison of different methods for determining FIR filter orders

Fig. 11. DFT comparison of unfiltered and filtered 25 concatenated cycles for operating point E1.1 at a.) 0.1°CA and b.) 1°CA. The red vertical line represents the stop-band frequency.

Fig. 12. DFT comparison of unfiltered and filtered 25 concatenated cycles for operating point E2.1 at a.) 0.1°CA and b.) 1°CA. The red vertical line represents the stop-band frequency.
5. Validation of methods based on ROHR traces

The quality of the ROHR trace is primarily dependent on the pressure and pressure derivative. As volume and volume derivative given by the geometrical data, and gas properties do not feature abrupt variations, they would be amplified in the evaluation of the ROHR trace as discussed in the introduction if large oscillations are present in the pressure trace. As ROHR generally illustrates the results of a combustion analysis on a system level, it will be used, in addition to the analyses in Sections 4.2 and 4.3, to finally validate the efficiency of the filter to suppress undesired oscillations.

5.1. ROHR evaluation

ROHR was calculated by the standard extended equations derived from the first law of thermodynamics. The analysis was performed for the high-pressure part of the cycle. To increase the accuracy of the results, ROHR evaluation considered the heat transfer to the combustion chamber walls and variable gas properties. When analyzing lean-operated diesel engines with high combustion efficiency, it is sufficient to consider dependency of internal energy and gas constant on the concentration of burned fuel and temperature as pressure dependency is much less pronounced at these operating parameters. With these dependencies, the first law of thermodynamics can be made explicit in ROHR as [27]

$$
\frac{dQ_{\text{com}}}{d\varphi} = \left(1 + \frac{1}{RA \partial T} \right) p \frac{dV}{d\varphi} + \left( u - \frac{T \partial u}{A \partial T} \right) dm
$$

$$
+ m \left( \frac{\partial u}{W_{FB}} - T \frac{\partial R}{\partial T} \frac{\partial w_{FB}}{RA} \right) d\varphi_{FB} + m \frac{T \partial u}{pA \partial T} dp - \frac{dQ_{\text{ht}}}{d\varphi}.
$$

In Eq. (17), the heat transfer coefficient of the combustion chamber walls was evaluated based on the correlation of Hohenberg heat transfer model [28], whereas the gas properties are given by input maps taking into account chemical equilibrium considerations. It must be noted that neither the use of Eq. (17) including gas property treatment nor the use of the applied heat transfer correlation limit the general applicability of the method.

5.2. Results

The ROHR analysis was performed on the operating points listed in Table 2 and Table 3. The main goal of this ROHR analysis is to illustrate the ability of the proposed workflow (Appendix
Cite paper as: RAŠIĆ, Davor, VIHAR, Rok, ŽVAR BAŠKOVIČ, Urban, KATRAŠNIK, Tomaz. Methodology for processing pressure traces used as inputs for combustion analyses in diesel engines. Measurement Science and Technology, Volume 28, Number 5 (2017), ISSN 0957-0233. doi: https://doi.org/10.1088/1361-6501/aa5f9e

B) to process in-cylinder pressure traces of a diesel internal combustion engine in a way that ensures high-quality ROHR traces, regardless of the engine type, operating point, or sampling frequency.

Fig. 13. ROHR comparison of unfiltered pressure trace and filtered pressure traces for operating point E1.1 at 0.1°C/CA sampling frequency. The amplitude in all subfigures is the heat release rate [J].

Fig. 14. ROHR comparison of unfiltered pressure trace and filtered pressure traces for
Fig. 15. ROHR comparison of unfiltered pressure trace and filtered pressure traces for operating point E1.1 at 1°CA sampling frequency. The amplitude in all subfigures is the heat release rate [J].

Fig. 16. ROHR comparison of unfiltered pressure trace and filtered pressure traces for
Figs. 13 and 14 show the comparison of ROHR traces based on the unfiltered pressure trace and filtered pressure traces for operating point E1.1 at 0.1°CA and 1°CA sampling frequencies. Figs. 15 and 16 show the comparison of ROHR traces based on the unfiltered pressure trace and filtered pressure traces for operating point E2.1 at 0.2°CA and 1°CA sampling frequencies. The ROHR traces confirm the trends indicated in the spectral analyses. It is evident from Figs. 13, 14, 15, and 16 that the ROHR traces, evaluated based on the unfiltered pressure traces, would feature large oscillations, and it is also expected that averaging will suppress these oscillations to some extent as these trends can be observed also in Figs. 6a, 7a, 8a, and 9a. Figs. 13, 14, 15, and 16 also confirm the results in Figs. 6b, 7b, 8b, and 9b illustrating that the Parks-McClellan algorithm for determining the FIR filter order is not very efficient in ensuring oscillation-free ROHR traces. Similarly, the results in Fig. 10 confirm that the ROHR traces evaluated based on the pressure traces processed with the FIR filter, where the filter order was determined using the proposed method, are characterized by superposition of no or nearly no unwanted oscillations. It is also important to note that the filter, designed based on the proposed method is capable of efficiently suppressing the unwanted oscillation also in the case of processing only a single pressure trace. This feature is very important for online or CLCC applications. The same conclusion can also be drawn for higher engine speeds and load points presented in Figs. 17, 18, 19, and 20. As already mentioned previously in the paper, these figures indicate that the presence of unwanted oscillations is even more pronounced in this operating point; thus ROHR traces based on the unfiltered pressure trace do not even provide a basic indication on the ROHR. A similar observation can be concluded for the Parks-McClellan algorithm of determining the FIR filter order, which at the sampling frequency of 1 °CA is not applicable for predicting even the basic shape of the ROHR. Unlike both previous cases, ROHR traces calculated based on the pressure traces, processed with the FIR filter where the filter order was determined using the proposed method, are characterized by superposition of no or nearly no unwanted oscillations.
Fig. 17. ROHR comparison of unfiltered pressure trace and filtered pressure traces for operating point E1.7 at 0.1°CA sampling frequency. The amplitude in all subfigures is the heat release rate [J].

Fig. 18. ROHR comparison of unfiltered pressure trace and filtered pressure traces for operating point E1.7 at 1°CA sampling frequency. The amplitude in all subfigures is the heat release rate [J].
Fig. 19. ROHR comparison of unfiltered pressure trace and filtered pressure traces for operating point E2.2 at 0.2°CA sampling frequency. The amplitude in all subfigures is the heat release rate [J].

Fig. 20. ROHR comparison of unfiltered pressure trace and filtered pressure traces for operating point E2.2 at 1°CA sampling frequency. The amplitude in all subfigures is the heat release rate [J].
6. Summary and Conclusion

In this paper, a novel methodology for designing an optimum equiripple FIR filter for processing in-cylinder pressure traces is proposed. The main objective is to derive the in-cylinder pressure traces that allow for calculation of high-quality ROHR traces. The entire novel methodology is presented in Appendix B. It is based on the innovative approach of determining the transition band frequencies and optimum filter order.

The transition band frequencies were determined using the DFT analysis, followed by a refinement step based on the STFT analysis. The refinement using the STFT analysis has proved to be a desirable and almost necessary aspect of accurately determining the transition band frequencies. In the analyzed case, it turned out that the transition band frequencies cannot be determined very accurately solely on the basis of the DFT analysis, as it captures the amplitude of the excited vibrational modes irrespective of their repeatability. On the other hand, the STFT analysis provides an insight into the frequency spectra of the lower and higher intensities of vibrational eigenmodes in the time domain, thus enabling determination of transition band frequencies with greater fidelity.

Furthermore, the paper proposes an innovative method of determining the optimum filter orders as it was proven that the existing methods do not allow a satisfactory level of filtering in calculating the ROHR traces. The method is based on the integral of the frequency spectrum between the stop-band frequency and the Nyquist frequency as well as on a robust method to find the optimum filter order.

It was demonstrated that an innovative approach of designing an FIR filter ensures inherent suppression of frequencies higher than the stop-band frequency. The spectrum analysis implies that newly designed filters are much more efficient in eliminating frequencies higher than the stop-band frequencies compared with the existing filters.

The novel workflow for designing an optimum FIR filter is validated also on a broad spectrum of ROHR traces. The filters proved to be efficient for both engines, at all sampling frequencies and in all operating points; therefore, the new proposed methodology demonstrated its efficacy and robustness. The robustness of the method is further demonstrated by maintaining all parameters of the method constant for both engines, for all operating points, and for all sampling frequencies as well as transition band interval sizes. However, the method is certainly completely open to application of other parameters and correlations.

The entire workflow is designed in a way that in can be implemented in a computer code, and thus used for the automatic design of the FIR filter. The innovative method is characterized by higher filter orders compared with existing methods of determining the FIR filter order. Based on the current state-of-the-art computer hardware, this is not a limitation for offline application, whereas this potential drawback for online application is also diminishing with the evolution of engine control units.

APPENDIX A. Engine specifications

Table A1. Engine 1 specifications

<table>
<thead>
<tr>
<th>Engine</th>
<th>Peugeot 1.6 HDi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinders</td>
<td>4, inline</td>
</tr>
<tr>
<td>Displacement</td>
<td>1560 cm³</td>
</tr>
<tr>
<td>Bore × stroke</td>
<td>75 mm × 88.3 mm</td>
</tr>
</tbody>
</table>
Cite paper as: RAŠIĆ, Davor, VIHAR, Rok, ŽVAR BAŠKOVIČ, Urban, KATRAŠNIK, Tomaž. Methodology for processing pressure traces used as inputs for combustion analyses in diesel engines. *Measurement Science and Technology, Volume 28, Number 5 (2017)*, ISSN 0957-0233. doi: https://doi.org/10.1088/1361-6501/aa5f9e

<table>
<thead>
<tr>
<th>Compression ratio</th>
<th>18:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel injection system</td>
<td>diesel common rail</td>
</tr>
<tr>
<td>Maximum power</td>
<td>$66.2 \text{ kW} @ 4000 \text{ min}^{-1}$</td>
</tr>
<tr>
<td>Maximum torque</td>
<td>$215 \text{ N} \cdot \text{m} @ 1750 \text{ min}^{-1}$</td>
</tr>
</tbody>
</table>

Table A2. Engine 2 specifications

<table>
<thead>
<tr>
<th>Engine</th>
<th>MAN D 0826 LOH 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinders</td>
<td>6, inline</td>
</tr>
<tr>
<td>Displacement</td>
<td>$6870 \text{ cm}^3$</td>
</tr>
<tr>
<td>Bore × stroke</td>
<td>$108 \text{ mm} \times 125 \text{ mm}$</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>18:1</td>
</tr>
<tr>
<td>Fuel injection system</td>
<td>Mechanically controlled direct injection</td>
</tr>
<tr>
<td>Maximum power</td>
<td>$162 \text{ kW} @ 2400 \text{ min}^{-1}$</td>
</tr>
<tr>
<td>Maximum torque</td>
<td>$825 \text{ N} \cdot \text{m} @ 1400–1700 \text{ min}^{-1}$</td>
</tr>
</tbody>
</table>
APPENDIX B. Flow chart for determining an optimum FIR filter order

Fig. B1. Flow chart for determining an optimum FIR filter order based on the proposed novel workflow. The dotted line indicates the transition between the methodology for determining transition band frequencies (TBF) that comprises innovative steps TBF 4.1 and TBF 4.2 (presented in Section 3.3) and the innovative methodology for determining an optimum FIR filter order (presented in Section 4.2).

APPENDIX C. Equations for FIR filter order estimation

Kaiser equation:
\[ N_K \cong \frac{-20 \log_{10}(\sqrt{\delta_p \delta_s}) - 13}{14.6 \left(\omega_s - \omega_p\right)/2\pi} \]  \hspace{1cm} (C. 1)

**Hermann equation:**

\[ N_H \cong \frac{D_\infty(\delta_p, \delta_s) - F(\delta_p, \delta_s) \left[\left(\frac{\omega_s - \omega_p}{2\pi}\right)^2\right]}{\left(\frac{\omega_s - \omega_p}{2\pi}\right)^2} \]  \hspace{1cm} (C. 2)

where

\[ D_\infty(\delta_p, \delta_s) = \left[ a_1 \left(\log_{10}(\delta_p)\right)^2 + a_2 \left(\log_{10}(\delta_p)\right) + a_3 \right] \log_{10}(\delta_s) + \left[ a_4 \left(\log_{10}(\delta_p)\right)^2 + a_5 \left(\log_{10}(\delta_p)\right) + a_6 \right] \]

\[ F(\delta_p, \delta_s) = b_1 + b_2 \left(\log_{10}(\delta_p) - \log_{10}(\delta_s)\right) \]

with

\[ a_1 = 0.005309, a_2 = 0.07114, a_3 = -0.4761, a_4 = 0.00266, a_5 = 0.5941, a_6 = 0.4278 \]

\[ b_1 = 11.01217, b_2 = 0.51244 \]

**Bellanger equation:**

\[ N_B \cong \frac{-20 \log_{10}(\sqrt{\delta_p \delta_s}) - 13}{14.6 \left(\omega_s - \omega_p\right)/2\pi} \]  \hspace{1cm} (C. 3)

**Parks-McClellan algorithm:**

Alternation theorem:

\[ W(\omega_i) [H_a(e^{j\omega_i}) - A_e(e^{j\omega_i})] = (-1)^i \delta \quad i = 1, 2, ..., (L + 2) \]  \hspace{1cm} (C. 4)

where \( \delta \) is the optimum error. With \( \omega_p \) and \( \omega_s \) fixed, an iterative algorithm finds the optimal approximation of \( A_e(e^{j\omega_i}) \).

Algorithm for the Parks-McClellan FIR filter design method:

1. Make an initial guess of \((L + 2)\) extremal frequencies \( \omega_0, \omega_1, ..., \omega_{L+1} \).
2. Solve for polynomial coefficients and \((L + 2)\) equations in \((L + 2)\) unknowns.
3. Evaluate \( A_e(e^{j\omega_i}) \) on a dense set of frequencies and choose a new set of extremal frequencies.
4. Check whether the extremal frequencies changed. If yes, proceed to step 2. If no, the algorithm has converged.
APPENDIX D. Dependency of function $P$ on sampling frequency and transition band interval size

The aim of this section is to analyze the shapes of function $P$ (Eq. (8)) for a large variety of parameters to provide reasoning on the method of finding the minimum of function $P$ outlined in Section 4.2. To support such an analysis, Fig. D1 shows the function $P$ at different FIR filter orders using various transition band interval sizes, while Fig. D2 shows the dependency of the function $P$ on the sampling frequency for different transition band interval sizes. Moreover, to ensure sufficient level of generality, the analysis was conducted for different operating points and for different sampling frequencies. The figures presented were generated by increasing the FIR filter orders in steps of 2. The analysis was performed for the entire pool of operating points; however, for brevity, the results for dependency of function $P$ on the transition band interval size are shown only for the operating point E1.1 at 0.1°CA and the results for dependency of function $P$ on the sampling frequency for various transition band interval sizes are shown only for operating points E1.1, E1.4, and E1.7.

The results of all analyzed cases indicate that with increasing FIR filter order, the value of the function $P$ is first decreasing (in between might feature some nondistinct local minima) until reaching a minimum value, after which, it first gradually increases with increasing of the FIR filter order, before reaching the area of filter instability. This behavior is crucial for the proposed innovative method as it can be observed that the global minima of the function $P$ lies in the continuous area of the function with ample margin towards the area of filter instability. It is also very important that qualitatively, the shape of the function $P$ is not dependent on sampling frequency, transition band interval size, or operating point, which is the case also for all the points that were not shown. This is valid despite the fact that the minimum value of the function $P$ and its position appear at different FIR filter orders for different sampling frequencies, transition band interval sizes, and operating points.

The pass-band frequency was a fixed value, while the stop-band frequency was increased. Using nonadequate transition band frequencies certainly hinders the location of the optimum FIR filter order; however, the main goal of this analysis is to demonstrate that the qualitative shape of the function $P$ is preserved. The results indicate that the FIR filter order location is strongly connected to the transition band interval size; specifically, with increasing the transition band interval size, the optimum FIR filter order is decreasing as observed in Fig. D1. Furthermore, the results also indicate that the FIR filter order location is connected to the sampling frequency. Fig. D2 highlights the correlation between the sampling frequency and the optimum FIR filter order. It can be observed that when the sampling frequency is increasing, the optimum FIR filter order is increasing as well. A similar phenomenon can be observed for the entire pool of operating points on both engines.
Fig. D1. Dependency of function $P$ on transition band interval size for operating point E1.1. Bold color circles represent the minimum value of function $P$ for each respective transition band interval size.
Fig. D2. Dependency of function $P$ on the sampling frequency for different operating points (sampling frequencies): a.) for transition band interval size 1000 Hz, b.) for transition band interval size 2000 Hz, c.) for transition band interval size 3000 Hz, and d.) for transition band interval size 4000 Hz. Bold color circles represent the minimum value of function $P$ for each respective operating point.
References


Methodology for processing pressure traces used as inputs for combustion analyses in diesel engines. *Measurement Science and Technology, Volume 28, Number 5 (2017), ISSN 0957-0233. doi: https://doi.org/10.1088/1361-6501/aa5f9e*


